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Connectivity-Preserving Swarm Teleoperation With A Tree Network

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Abstract— **During swarm teleoperation, the operator may threaten the distance-dependent inter-robot communications and, with them, the connectivity of the slave swarm. To prevent the operator from disconnecting the swarm, this paper develops a constructive strategy to dynamically modulate the interconnections of, and the local damping injections at, all slave robots. Lyapunov-based set invariance analysis shows that the strategy preserves all interaction links in the tree network while synchronizing the slave swarm. By properly limiting the impact of the user command rather than rejecting it entirely, the proposed explicit gain update law enables the operator to guide the motion of the slave swarm to the extent to which it does not endanger swarm connectivity. An experiment illustrates that the proposed strategy can maintain the tree network connectivity of a teleoperated swarm.**

I. Introduction

Compared to autonomous multi-robot systems (MRS-s), semi-autonomous teleoperated swarms are partially controlled by human operators and, thus, better suited for complex tasks in unpredictable environments [1]. Existing swarm teleoperation work has connected the master and slave swarm via velocity-like variables [2], [3], steered bearing formations [4], decoupled the master and slave swarm via virtual kinematic points [5], improved the agility of human-swarm interaction for optimal coverage control via time-varying density functions [6], flown arbitrarily many aerial robots with collision avoidance [7], exchanged forces between an operator and a group of unmanned aerial vehicles [8].

In practice, the inter-robot communications needed for distributed MRS synchronization are constrained by the inter-robot distances [2]. Robot teams with limited communication range need coordination strategies with guaranteed connectivity. For semi-autonomous MRS-s, passivity-based control maintains both global [2], [9] and local [5] connectivity. The gradient laws that preserve global connectivity [2] and regulate the degree of connectivity [9] derive from a function of the estimated algebraic connectivity of the teleoperated swarm [2]. Since the algorithm [10] estimates the algebraic connectivity, its accuracy and convergence rate determine the effectiveness of those gradient laws and thus the safety of the swarm [11]. The estimation errors foil the ability to mitigate perturbations of global connectivity control even for first-order MRS-s [12]. Controls derived from a function of distances between virtual kinematic points [5] provably maintain only connectivity. Connectivity preservation through infinitesimal formation rigidity can be sought using range [13], [14] or bearing [15] measurements. Their comparative review [16] illustrates applications of bearing rigidity in formation control and network localization.

This paper contributes a constructive dynamic coupling and damping injection law for connectivity-preserving swarm teleoperation with a tree network. Without loss of generality, the design assumes that only one slave robot receives the user command from the master. A customized potential function of inter-robot distances enables set invariance analysis which proves that proper upper bounding of the energy stored in the slave swarm guarantees its local connectivity. The structural controllability of a tree network yields decentralized controls that provably bound the swarm energy. The explicit law for updating the control gains is derived (i) by transforming the swarm dynamics into a first-order representation with state-dependent mismatches via sliding surfaces and (ii) by suppressing the impact of those mismatches on connectivity via dynamically modulating the interconnections and local damping injections of all slave robots based on the inter-slave distances. Effectively, the proposed strategy limits the user-injected energy into the tree network thereby preventing the operator from disconnecting the slave swarm. An experiment with a group of haptic robots validates the preservation of the tree network of a teleoperated swarm.

II. Problem Formulation

Consider a robot swarm teleoperation system with one master and N slaves, one of which is an informed slave and communicates with the master unconstrained by distance [5]. Let the inter-slave communications be undirected and constrained by the communication distance r. A user operates the master to command the slave group to a desired location. Assuming passive master-informed slave coupling, this paper develops a controller to preserve the connectivity of the slave swarm under the user command sent by the master.

Let the slave swarm be a network of n-degree-of-freedom (n-DOF) Euler-Lagrange (EL) systems:

$$\begin{aligned}\mathbf{M}_1(\mathbf{x}_1)\ddot{\mathbf{x}}_1 + \mathbf{C}_1(\mathbf{x}_1,\dot{\mathbf{x}}_1)\dot{\mathbf{x}}_1 &= \mathbf{u}_1 + \mathbf{f},\\ \mathbf{M}_s(\mathbf{x}_s)\ddot{\mathbf{x}}_s + \mathbf{C}_s(\mathbf{x}_s,\dot{\mathbf{x}}_s)\dot{\mathbf{x}}_s &= \mathbf{u}_s.\end{aligned} \tag{1}$$

In Equation (1): the subscript 1 indicates the informed slave that receives the user command from the master; subscripts $s = 2, \cdots, N$ index the remaining $N-1$ slaves; and $\mathbf{f}$ is the time-varying command from the master. For each slave $i = 1, \cdots, N$: $\mathbf{x}_i$, $\dot{\mathbf{x}}_i$ and $\ddot{\mathbf{x}}_i$ are its position, velocity and acceleration; $\mathbf{M}_i(\mathbf{x}_i)$ and $\mathbf{C}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i)$ are its matrices of inertia and of Coriolis and centrifugal effects; and $\mathbf{u}_i$ is its connectivity-preserving control force to be designed. The dynamics (1) have the following properties:

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P.1 The inertia matrices $\mathbf{M}_i(\mathbf{x}_i)$ are symmetric, positive definite and uniformly bounded by: $\lambda_{i1}\mathbf{I} \preceq \mathbf{M}_i(\mathbf{x}_i) \preceq \lambda_{i2}\mathbf{I}$ for any $\mathbf{x}_i \in \mathbb{R}^n$, where $\lambda_{i1} > 0$ and $\lambda_{i2} > 0$;

P.2 $\dot{\mathbf{M}}_i(\mathbf{x}_i) - 2\mathbf{C}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i)$ are skew-symmetric;

P.3 $\mathbf{C}_i(\mathbf{x}_i, \mathbf{y}_i)$ are linear in $\mathbf{y}_i$, and there exist $c_i > 0$ such that $\|\mathbf{C}_i(\mathbf{x}_i, \mathbf{y}_i)\mathbf{z}_i\| \leq c_i\|\mathbf{y}_i\|\|\mathbf{z}_i\|$, $\forall \mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i \in \mathbb{R}^n$.

Because all inter-slave communications are constrained by the same radius r, slaves i and j can exchange information at time $t \geq 0$ iff their distance is strictly smaller than r, i.e., $\|\mathbf{x}_{ij}\| = \|\mathbf{x}_i(t) - \mathbf{x}_j(t)\| < r$, $\forall i, j \in \{1, \cdots, N\}$. Then, this paper preserves the connectivity of the slave swarm by maintaining certain bidirectional links (i, j) through properly constraining the respective $\|\mathbf{x}_{ij}(t)\|$ for all time $t \geq 0$.

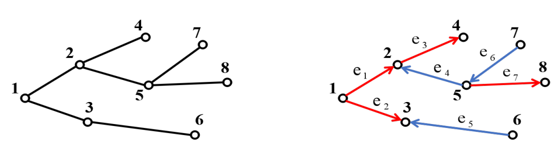
Let an undirected graph $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$ represent the inter-slave information exchanges. The vertex set $\mathcal{V} = \{1, \cdots, N\}$ collects all the slaves in the swarm. The edge set $\mathcal{E}(t) \subset \{(i, j) \in \mathcal{V} \times \mathcal{V}\}$ includes all inter-slave communication links. By definition, robots i and j are adjacent, i.e., $(i, j) \in \mathcal{E}(t)$, iff they exchange information. The set of neighbours of slave $i \in \mathcal{V}$ is $\mathcal{N}_i(t) = \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}(t)\}$ and collects all slaves adjacent to it at time $t \geq 0$. In $\mathcal{G}(t)$, a path between slaves i and j is a sequence of distinct vertices $i, a, b, \cdots, j$ such that consecutive vertices are adjacent. The graph $\mathcal{G}(t)$ is connected iff a path exists between any two distinct vertices. Further, $\mathcal{G}(t)$ is a tree if any two vertices are connected by exactly one path, as in Figure 1(a).

Given a tree $\mathcal{G}(t)$ of order N, its associated weighted adjacency matrix $\mathbf{A} = [a_{ij}]$ is defined by: $a_{ij} = a_{ji} > 0$ if $(i, j) \in \mathcal{E}(t)$, and $a_{ij} = 0$ otherwise. Its weighted Laplacian matrix $\mathbf{L} = [l_{ij}]$ is defined by: $l_{ij} = \sum_{k \in \mathcal{N}_i(t)} a_{ik}$ if $j = i$, and $l_{ij} = -a_{ij}$ otherwise. If $a_{ij} \in \{0, 1\}$, the graph has an unweighted Laplacian matrix $\overline{\mathbf{L}}$. Let an orientation of $\mathcal{G}(t)$ define an oriented graph $\mathcal{G}^*(t)$, and label each oriented edge (i, j) as e_k, $k = 1, \cdots, N-1$, with weight $w(e_k) = a_{ij} = a_{ji}$, as shown in Figure 1(b). The incidence matrix $\mathbf{D} = [d_{hk}]$ associated to $\mathcal{G}^*(t)$ is defined by: $d_{hk} = 1$ if vertex h is the head of edge e_k; $d_{hk} = -1$ if h is the tail of e_k and $d_{hk} = 0$ otherwise. The edge Laplacian of $\mathcal{G}^*(t)$ is then $\mathbf{L}_e = \mathbf{D}^\mathsf{T}\mathbf{D}$. The following lemmas [17] will help prove Lemma 1 in Section III:

L.1 The second smallest eigenvalue λ_L of the unweighted Laplacian $\overline{\mathbf{L}}$ is positive, i.e., $\lambda_L > 0$.

L.2 $\mathbf{L}_e$ and $\overline{\mathbf{L}}$ have the same set of nonzero eigenvalues.

L.3 The weighted Laplacian admits the decomposition $\mathbf{L} = \mathbf{D}\mathbf{W}\mathbf{D}^\mathsf{T}$, with $\mathbf{W}$ an $(N-1)\times(N-1)$ diagonal matrix with $w(e_k)$, $k = 1, \cdots, N-1$, on the diagonal.



(a) The tree network $\mathcal{G}(t)$ of a slave swarm.

(b) An orientation $\mathcal{G}^*(t)$ of the undirected tree network $\mathcal{G}(t)$.

Fig. 1. The tree network $\mathcal{G}(t)$ and an orientation $\mathcal{G}^*(t)$ for an illustrative slave swarm in (1).

The paper adopts the following assumptions on the initial system configuration and on the command from the master.

Assumption 1. *The initial interaction network $\mathcal{G}(0)$ of the slave swarm is a tree with all initially adjacent robots strictly within communication distance, i.e., $\forall (i, j) \in \mathcal{E}(0)$, $\exists \epsilon > 0$ such that $\|\mathbf{x}_{ij}(0)\| < r - \epsilon$.*

Assumption 2. *The user command from the master side is bounded by $\|\mathbf{f}\| \leq \overline{f}$.*

Since a connected graph has at least one spanning tree [17], Assumption 1 directly presumes a minimally connected initial interaction network $\mathcal{G}(0)$. Since the EL slaves have inertia, it adopts the same distance condition $\|\mathbf{x}_{ij}(0)\| < r - \epsilon$ on initially adjacent robots $(i, j) \in \mathcal{E}(0)$ as in connectivity preservation of fully autonomous second-order MRS-s [18]. Then, this paper addresses the following connectivity-preserving swarm teleoperation problem:

Problem 1. *Find distributed control laws to drive the teleoperated swarm* (1) *satisfying Assumptions 1 and 2 such that:*

1. *The velocities of, and the position errors between, any two slave robots $i, j = 1, \cdots, N$ are bounded in the presence of the user command, i.e., $\{\dot{\mathbf{x}}_i, \dot{\mathbf{x}}_j, \mathbf{x}_i - \mathbf{x}_j\} \in \mathcal{L}_\infty$ when $\mathbf{f} \neq \mathbf{0}$;*
2. *All slave robots $i, j = 1, \cdots, N$ are synchronized in the absence of the user command, i.e., $\{\dot{\mathbf{x}}_i, \dot{\mathbf{x}}_j, \mathbf{x}_i - \mathbf{x}_j\} \rightarrow \mathbf{0}$ when $\mathbf{f} = \mathbf{0}$;*
3. *All interaction links $(i, j) \in \mathcal{E}(0)$ of the initial slave network $\mathcal{G}(0)$ are maintained, i.e., $\forall (i, j) \in \mathcal{E}(0) \Rightarrow (i, j) \in \mathcal{E}(t)$ $\forall t \geq 0$, and, with them, the connectivity of the slave swarm $\mathcal{G}(t)$ is preserved.*

The first two objectives in Problem 1 are similar to those of conventional bilateral teleoperation [19]. The last objective and Assumption 1 show that the proposed strategy preserves connectivity by maintaining a spanning tree of the teleoperated swarm. Future research will study switching spanning trees for connectivity-preserving swarm teleoperation.

III. Main Results

By Assumption 1 and the third objective in Problem 1, this paper maintains the connectivity of the teleoperated swarm by rendering invariant the edge set $\mathcal{E}(t)$ of the tree network $\mathcal{G}(0)$ for any $t \geq 0$. The inter-slave communication links $(i, j) \in \mathcal{E}(t)$ are verified by:

$$\psi(\|\mathbf{x}_{ij}\|) = \frac{P\|\mathbf{x}_{ij}\|^2}{r^2 - \|\mathbf{x}_{ij}\|^2 + Q}, \tag{2}$$

where P and Q are positive constants to be designed. Being continuous, positive and strictly increasing with respect to $\|\mathbf{x}_{ij}\|$ for any $\|\mathbf{x}_{ij}\| \in [0, r]$ [18], the functions $\psi(\|\mathbf{x}_{ij}\|)$ can describe the energy stored in all links $(i, j) \in \mathcal{E}(0)$ by:

$$V_p = \frac{1}{2}\sum_{i=1}^{N}\sum_{j \in \mathcal{N}_i(0)} \psi(\|\mathbf{x}_{ij}\|). \tag{3}$$

The following proposition guarantees the feasibility of connectivity preservation using functions (2):

Proposition 1. *Under Assumption 1, V_p in* (3) *satisfies:*

$$V_p(0) + \Delta < \frac{Pr^2}{Q} = \psi_{\max}$$

for any $\Delta > 0$ and any Q and P selected by:

$$\begin{aligned} &\left[r^2 - (N-1)(r-\epsilon)^2\right] Q + \left[r^2 - (r-\epsilon)^2\right] r^2 > 0, \\ &P > \frac{\left[r^2 - (r-\epsilon)^2 + Q\right] Q \cdot \Delta}{\left[r^2 - (r-\epsilon)^2 + Q\right] r^2 - (N-1)Q \cdot (r-\epsilon)^2}. \end{aligned} \tag{4}$$

In turn, Proposition 1 permits to examine the distance constraint on every link $(i, j) \in \mathcal{E}(0)$ by Proposition 2:

Proposition 2. *Given Assumption 1, $\Delta > 0$, and P and Q selected by* (4)*, let V_p in* (3) *satisfy:*

$$V_p(\tau) \leq V_p(0) + \Delta, \quad \forall \tau \in [0, t]. \tag{5}$$

Then $\|\mathbf{x}_{ij}(t)\| < r$ for any $(i, j) \in \mathcal{E}(0)$.

Propositions 1 and 2 are proven in [20]. They indicate that set invariance [21], [22] is key to proving connectivity preservation in this paper: the paper constrains the distance $\|\mathbf{x}_{ij}\|$ between each pair of initially adjacent slaves $(i, j) \in \mathcal{E}(0)$ to render invariant the edge set $\mathcal{E}(t)$ of a teleoperated swarm with a tree network. The potential functions (2) and (3) characterize the inter-slave distances in Propositions 1 and 2 and thus serve to investigate the invariance of $\mathcal{E}(t)$. A similar strategy has been used in [18] to prove connectivity maintenance for autonomous double-integrator multi-agent systems. In contrast, this paper preserves the tree network of a robotic slave swarm driven by a time-varying and unpredictable user command. The main contribution is the constructive design of a dynamic coupling and damping injection controller that guarantees (5) under the perturbation of the user input transmitted by the master command $\mathbf{f}$.

To bound the potential energy V_p using local information of each robot i, define a surface $\mathbf{s}_i$ for each slave i by:

$$\mathbf{s}_i = \dot{\mathbf{x}}_i + \sigma \boldsymbol{\theta}_i, \tag{6}$$

where $i = 1, \cdots, N$, $\sigma > 0$ and:

$$\boldsymbol{\theta}_i = \sum_{j \in \mathcal{N}_i(0)} \nabla_i \psi(\|\mathbf{x}_{ij}\|) \tag{7}$$

with the gradient of $\psi(\|\mathbf{x}_{ij}\|)$ with respect to $\mathbf{x}_i$ given by:

$$\nabla_i \psi(\|\mathbf{x}_{ij}\|) = \frac{2P\left(r^2 + Q\right)}{\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^2} \left(\mathbf{x}_i - \mathbf{x}_j\right). \tag{8}$$

Then, the dynamics of the teleoperated swarm (1) can be transformed into:

$$\begin{aligned} \mathbf{M}_1(\mathbf{x}_1)\dot{\mathbf{s}}_1 + \mathbf{C}_1(\mathbf{x}_1, \dot{\mathbf{x}}_1)\mathbf{s}_1 &= \sigma \boldsymbol{\Delta}_1 + \mathbf{u}_1 + \mathbf{f}, \\ \mathbf{M}_s(\mathbf{x}_s)\dot{\mathbf{s}}_s + \mathbf{C}_s(\mathbf{x}_s, \dot{\mathbf{x}}_s)\mathbf{s}_s &= \sigma \boldsymbol{\Delta}_s + \mathbf{u}_s \end{aligned} \tag{9}$$

where $\boldsymbol{\Delta}_i$ are state-dependent mismatches given by:

$$\boldsymbol{\Delta}_i = \mathbf{M}_i(\mathbf{x}_i)\dot{\boldsymbol{\theta}}_i + \mathbf{C}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i)\boldsymbol{\theta}_i. \tag{10}$$

The following lemma is key to the connectivity maintenance proof in the remainder of the paper.

Lemma 1. *Given the teleoperated swarm* (1) *with the tree communications network $\mathcal{G}(0)$, the following holds:*

$$\sum_{i=1}^N \boldsymbol{\theta}_i^\mathsf{T} \boldsymbol{\theta}_i \geq \frac{4\lambda_L P}{r^2 + Q} V_p. \tag{11}$$

Proof. Define the weighted adjacency matrix $\mathbf{A} = [a_{ij}]$ associated with the tree $\mathcal{G}(0)$ by:

$$a_{ij} = \begin{cases} \frac{2P(r^2+Q)}{(r^2 - \|\mathbf{x}_{ij}\|^2 + Q)^2} & \text{if } j \in \mathcal{N}_i(0), \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding weighted laplacian $\mathbf{L} = [l_{ij}]$ is given by:

$$l_{ij} = \begin{cases} -a_{ij} & \text{if } j \neq i \\ \sum_{k \in \mathcal{N}_i(0)} a_{ik} & \text{else if } j = i. \end{cases}$$

Let $\mathbf{l}_i$ be the i-th row of $\mathbf{L}$ and $\mathbf{x} = [\mathbf{x}_1^\mathsf{T} \ \cdots \ \mathbf{x}_N^\mathsf{T}]^\mathsf{T}$. Then, it follows that:

$$\sum_{j \in \mathcal{N}_i(0)} \nabla_i \psi(\|\mathbf{x}_{ij}\|) = \left(\mathbf{l}_i \otimes \mathbf{I}_n\right) \mathbf{x}.$$

By the definition of $\boldsymbol{\theta}_i$ (7), the left-hand side of (11) becomes:

$$\begin{aligned} \sum_{i=1}^N \boldsymbol{\theta}_i^\mathsf{T} \boldsymbol{\theta}_i &= \sum_{i=1}^N \mathbf{x}^\mathsf{T} \left(\mathbf{l}_i \otimes \mathbf{I}_n\right)^\mathsf{T} \left(\mathbf{l}_i \otimes \mathbf{I}_n\right) \mathbf{x} \\ &= \mathbf{x}^\mathsf{T} \left[\left(\sum_{i=1}^N \mathbf{l}_i^\mathsf{T} \mathbf{l}_i\right) \otimes \mathbf{I}_n\right] \mathbf{x} = \mathbf{x}^\mathsf{T} \left(\mathbf{L}^\mathsf{T} \mathbf{L} \otimes \mathbf{I}_n\right) \mathbf{x}, \end{aligned}$$

and further, by Lemma L.3 and the definition of $\mathbf{W}$ in [17]:

$$\begin{aligned} &\sum_{i=1}^N \boldsymbol{\theta}_i^\mathsf{T} \boldsymbol{\theta}_i = \mathbf{x}^\mathsf{T} \left[\mathbf{D}\mathbf{W}\mathbf{D}^\mathsf{T}\mathbf{D}\mathbf{W}\mathbf{D}^\mathsf{T} \otimes \mathbf{I}_n\right] \mathbf{x} \\ &= \left[\left(\mathbf{W}\mathbf{D}^\mathsf{T} \otimes \mathbf{I}_n\right) \mathbf{x}\right]^\mathsf{T} \left(\mathbf{D}^\mathsf{T}\mathbf{D} \otimes \mathbf{I}_n\right) \left[\left(\mathbf{W}\mathbf{D}^\mathsf{T} \otimes \mathbf{I}_n\right) \mathbf{x}\right] \\ &= \overline{\mathbf{x}}^\mathsf{T} \left(\mathbf{L}_e \otimes \mathbf{I}_n\right) \overline{\mathbf{x}}, \end{aligned} \tag{12}$$

where $\overline{\mathbf{x}} = [\overline{\mathbf{x}}_1^\mathsf{T} \ \cdots \ \overline{\mathbf{x}}_{N-1}^\mathsf{T}]^\mathsf{T} = (\mathbf{W}\mathbf{D}^\mathsf{T} \otimes \mathbf{I}_n)\mathbf{x}$, with $\overline{\mathbf{x}}_k = \nabla_i \psi(\|\mathbf{x}_{ij}\|)$ for $e_k = (i, j)$, $k = 1, \cdots, N-1$, stacks the weighted position mismatches between all pairs of initially adjacent slaves $(i, j) \in \mathcal{E}(0)$ [17].

By Lemma L.1 [17], Assumption 1 implies that the unweighted laplacian $\overline{\mathbf{L}}$ has positive second smallest eigenvalue $\lambda_L > 0$. Because $\mathcal{G}(0)$ is a tree, $\mathbf{L}_e$ is an $(N-1) \times (N-1)$ matrix. By Lemma L.2 [17], its smallest eigenvalue is λ_L. Hence, (12), together with (8) and (2), lower-bounds the left-hand side of (11) by:

$$\begin{aligned} &\sum_{i=1}^N \boldsymbol{\theta}_i^\mathsf{T} \boldsymbol{\theta}_i \geq \lambda_L \overline{\mathbf{x}}^\mathsf{T} \overline{\mathbf{x}} = \lambda_L \sum_{(i,j) \in \mathcal{E}(0)} \left\|\nabla_i \psi(\|\mathbf{x}_{ij}\|)\right\|^2 \\ &= \sum_{(i,j) \in \mathcal{E}(0)} \frac{4\lambda_L P^2 (r^2 + Q)^2}{\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^4} \|\mathbf{x}_i - \mathbf{x}_j\|^2 \\ &= \sum_{(i,j) \in \mathcal{E}(0)} \frac{4\lambda_L P (r^2 + Q)^2}{\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^3} \psi(\|\mathbf{x}_{ij}\|) \geq \frac{4\lambda_L P}{r^2 + Q} V_p, \end{aligned}$$

■

In the teleoperated robot swarm network (1), each slave i receives information from its initial neighbours $j \in \mathcal{N}_i(0)$ at all time if $\|\mathbf{x}_{ij}(t)\| < r\ \forall t \geq 0$. To prove that controls based on $\psi(\|\mathbf{x}_{ij}\|)$ in (2) and V_p in (3) guarantee this condition and, with it, connectivity maintenance, this paper uses induction on time [18]: assume that all links $(i,j) \in \mathcal{E}(0)$ have been maintained during the time interval $[0,t)$, i.e., $\|\mathbf{x}_{ij}(\tau)\| < r$ $\forall \tau \in [0,t)$ and $\forall (i,j) \in \mathcal{E}(0)$; then use the position $\mathbf{x}_j(t)$ of slave $j \in \mathcal{N}_i(0)$ in the control of slave i at time t to prove that $\|\mathbf{x}_{ij}(t)\| < r\ \forall (i,j) \in \mathcal{E}(0)$ by Proposition 2. The controls proposed to render the edge set $\mathcal{E}(0)$ positively invariant are:

$$\mathbf{u}_i = -K_i(t)\mathbf{s}_i - D_i\dot{\mathbf{x}}_i - B_i\boldsymbol{\theta}_i, \quad i = 1, \cdots, N \tag{13}$$

with $K_i(t)$, D_i and B_i are positive gains to be determined.

Remark 1. By the definition of $\mathbf{s}_i$ in (6), the control $\mathbf{u}_i$ is:

$$\mathbf{u}_i = -\left[\sigma K_i(t) + B_i\right]\boldsymbol{\theta}_i - \left[K_i(t) + D_i\right]\dot{\mathbf{x}}_i,$$

where the first and second terms are coupling and damping injection forces, and $K_i(t)$ is state-dependent and thus time-varying. Specifically, $K_i(t)$ is updated based on the distances $\|\mathbf{x}_{ij}\|$ between slave i and its neighbours $j \in \mathcal{N}_i(0)$.

The Lyapunov candidate for connectivity preservation is:

$$V = \frac{1}{2}\sum_{i=1}^{N}\frac{1}{B_i + \sigma D_i}\mathbf{s}_i^{\mathsf{T}}\mathbf{M}_i(\mathbf{x}_i)\mathbf{s}_i + V_p, \tag{14}$$

with V_p defined in (3). Along the transformed dynamics (9) in closed-loop with the control (13), the derivative of V is:

$$\begin{aligned}\dot{V} =& \frac{1}{2}\sum_{i=1}^{N}\frac{1}{B_i + \sigma D_i}\left[\mathbf{s}_i^{\mathsf{T}}\dot{\mathbf{M}}_i(\mathbf{x}_i)\mathbf{s}_i + 2\mathbf{s}_i^{\mathsf{T}}\mathbf{M}_i(\mathbf{x}_i)\dot{\mathbf{s}}_i\right] \\ &+ \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)}\left[\dot{\mathbf{x}}_i^{\mathsf{T}}\nabla_i\psi(\|\mathbf{x}_{ij}\|) + \dot{\mathbf{x}}_j^{\mathsf{T}}\nabla_j\psi(\|\mathbf{x}_{ij}\|)\right] \\ =& \sum_{i=1}^{N}\frac{1}{B_i + \sigma D_i}\left[\sigma\mathbf{s}_i^{\mathsf{T}}\boldsymbol{\Delta}_i - K_i(t)\mathbf{s}_i^{\mathsf{T}}\mathbf{s}_i\right] + \frac{\mathbf{s}_1^{\mathsf{T}}\mathbf{f}}{B_1 + \sigma D_1} \\ &- \sum_{i=1}^{N}\frac{\mathbf{s}_i^{\mathsf{T}}\left(D_i\dot{\mathbf{x}}_i + B_i\boldsymbol{\theta}_i\right)}{B_i + \sigma D_i} + \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)}\dot{\mathbf{x}}_i^{\mathsf{T}}\nabla_i\psi(\|\mathbf{x}_{ij}\|),\end{aligned}$$

where Properties P.1 and P.2 of (1) and Assumption 1 have been applied. The definition of $\mathbf{s}_i$ in (6) and the use of $\boldsymbol{\theta}_i$ in (7) in the control $\mathbf{u}_i$ at time instant t, permitted by the assumption that $\mathcal{E}(\tau) = \mathcal{E}(0)$ for all $\tau \in [0,t)$, leads to:

$$\begin{aligned}&\mathbf{s}_i^{\mathsf{T}}(D_i\dot{\mathbf{x}}_i + B_i\boldsymbol{\theta}_i) \\ =& D_i\dot{\mathbf{x}}_i^{\mathsf{T}}\dot{\mathbf{x}}_i + \sigma D_i\dot{\mathbf{x}}_i^{\mathsf{T}}\boldsymbol{\theta}_i + B_i\dot{\mathbf{x}}_i^{\mathsf{T}}\boldsymbol{\theta}_i + \sigma B_i\boldsymbol{\theta}_i^{\mathsf{T}}\boldsymbol{\theta}_i \\ =& D_i\dot{\mathbf{x}}_i^{\mathsf{T}}\dot{\mathbf{x}}_i + \sigma B_i\boldsymbol{\theta}_i^{\mathsf{T}}\boldsymbol{\theta}_i + (B_i + \sigma D_i)\sum_{j\in\mathcal{N}_i(0)}\dot{\mathbf{x}}_i^{\mathsf{T}}\nabla_i\psi(\|\mathbf{x}_{ij}\|),\end{aligned}$$

and further to:

$$\begin{aligned}\dot{V} =& \sum_{i=1}^{N}\frac{\sigma\mathbf{s}_i^{\mathsf{T}}\boldsymbol{\Delta}_i - \sigma B_i\boldsymbol{\theta}_i^{\mathsf{T}}\boldsymbol{\theta}_i}{B_i + \sigma D_i} + \frac{\mathbf{s}_1^{\mathsf{T}}\mathbf{f}}{B_1 + \sigma D_1} \\ &- \sum_{i=1}^{N}\frac{1}{B_i + \sigma D_i}\left[K_i(t)\mathbf{s}_i^{\mathsf{T}}\mathbf{s}_i + D_i\dot{\mathbf{x}}_i^{\mathsf{T}}\dot{\mathbf{x}}_i\right].\end{aligned} \tag{15}$$

By its definition in (7), the derivative of $\boldsymbol{\theta}_i$ is:

$$\begin{aligned}\dot{\boldsymbol{\theta}}_i =& \sum_{j\in\mathcal{N}_i(0)}\frac{8P(r^2+Q)\mathbf{x}_{ij}^{\mathsf{T}}\dot{\mathbf{x}}_{ij}\mathbf{x}_{ij}}{\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^3} \\ &+ \sum_{j\in\mathcal{N}_i(0)}\frac{2P(r^2+Q)(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_j)}{\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^2},\end{aligned} \tag{16}$$

and algebraic manipulations using Properties P.1 and P.3 of (1) imply that:

$$\begin{aligned}\mathbf{s}_i^{\mathsf{T}}\mathbf{M}_i(\mathbf{x}_i)\dot{\boldsymbol{\theta}}_i \leq& \sum_{j\in\mathcal{N}_i(0)}\left[2\left(\eta_i + \gamma_i\right)\left(\dot{\mathbf{x}}_i^{\mathsf{T}}\dot{\mathbf{x}}_i + \dot{\mathbf{x}}_j^{\mathsf{T}}\dot{\mathbf{x}}_j\right)\right] \\ &+ \sum_{j\in\mathcal{N}_i(0)}\left[\frac{16\lambda_{i2}^2P^2(r^2+Q)^2\|\mathbf{x}_{ij}\|^4}{\eta_i\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^6}\mathbf{s}_i^{\mathsf{T}}\mathbf{s}_i\right. \\ &\left.+ \frac{\lambda_{i2}^2P^2\left(r^2+Q\right)^2}{\gamma_i\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^4}\mathbf{s}_i^{\mathsf{T}}\mathbf{s}_i\right]\end{aligned} \tag{17}$$

with $\eta_i > 0$ and $\gamma_i > 0$, and that:

$$\begin{aligned}&\mathbf{s}_i^{\mathsf{T}}\mathbf{C}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i)\boldsymbol{\theta}_i \\ \leq& \sum_{j\in\mathcal{N}_i(0)}\left[\frac{c_i^2P^2\left(r^2+Q\right)^2\|\mathbf{x}_{ij}\|^2}{2\zeta_i\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^4}\mathbf{s}_i^{\mathsf{T}}\mathbf{s}_i + 2\zeta_i\dot{\mathbf{x}}_i^{\mathsf{T}}\dot{\mathbf{x}}_i\right]\end{aligned} \tag{18}$$

with $\zeta_i > 0$. Hence, the impact of the mismatch $\boldsymbol{\Delta}_i$ given in (10) can be upper-bounded by:

$$\begin{aligned}\mathbf{s}_i^{\mathsf{T}}\boldsymbol{\Delta}_i \leq& \sum_{j\in\mathcal{N}_i(0)}\Big[\Lambda_{ij}(t)\mathbf{s}_i^{\mathsf{T}}\mathbf{s}_i + 2(\eta_i + \gamma_i)\dot{\mathbf{x}}_j^{\mathsf{T}}\dot{\mathbf{x}}_j \\ &+ 2(\eta_i + \gamma_i + \zeta_i)\dot{\mathbf{x}}_i^{\mathsf{T}}\dot{\mathbf{x}}_i\Big],\end{aligned} \tag{19}$$

where:

$$\begin{aligned}\Lambda_{ij}(t) =& \frac{16\lambda_{i2}^2P^2(r^2+Q)^2\|\mathbf{x}_{ij}\|^4}{\eta_i\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^6} \\ &+ \frac{\lambda_{i2}^2P^2\left(r^2+Q\right)^2}{\gamma_i\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^4} + \frac{c_i^2P^2\left(r^2+Q\right)^2\|\mathbf{x}_{ij}\|^2}{2\zeta_i\left(r^2 - \|\mathbf{x}_{ij}\|^2 + Q\right)^4}.\end{aligned}$$

The user-injected energy can be measured by:

$$\frac{\mathbf{s}_1^{\mathsf{T}}\mathbf{f}}{B_1 + \sigma D_1} \leq \frac{1}{4\Gamma}\|\mathbf{f}\|^2 + \frac{\Gamma}{\left(B_1 + \sigma D_1\right)^2}\mathbf{s}_1^{\mathsf{T}}\mathbf{s}_1, \tag{20}$$

where $\Gamma > 0$. Then, after substitution from (19) and (20) in (15), $\dot{V}$ can be upper-bounded by:

$$\dot{V} \leq -\sum_{i=1}^{N}\frac{\overline{K}_i(t)\mathbf{s}_i^{\mathsf{T}}\mathbf{s}_i + \overline{D}_i\dot{\mathbf{x}}_i^{\mathsf{T}}\dot{\mathbf{x}}_i + \sigma B_i\boldsymbol{\theta}_i^{\mathsf{T}}\boldsymbol{\theta}_i}{B_i + \sigma D_i} + \frac{\|\mathbf{f}\|^2}{4\Gamma}, \tag{21}$$

where:

$$\begin{aligned}\overline{K}_i(t) =& K_i(t) - \sigma\sum_{j\in\mathcal{N}_i(0)}\Lambda_{ij}(t) - \frac{\Gamma_i}{\left(B_1 + \sigma D_1\right)^2} \\ \overline{D}_i =& D_i - 2\sigma\sum_{j\in\mathcal{N}_i(0)}\left(\eta_i + \gamma_i + \zeta_i + \eta_j + \gamma_j\right)\end{aligned}, \tag{22}$$

with $\Gamma_i = \Gamma$ if $i = 1$ and $\Gamma_i = 0$ otherwise.

With Lemma 1, the invariance of $\mathcal{E}(0)$ and, with it, connectivity maintenance for the teleoperated swarm (1) under the controls (13) is guaranteed by the following theorem:

Theorem 1. *The controls* (13) *maintain the connectivity of the teleoperated swarm* (1) *with Assumptions 1 and 2 by rendering the edge set* $\mathcal{E}(0)$ *invariant if their parameters are selected as follows for all slaves* $i = 1, \cdots, N$*:*

1. *choose* ρ, σ, η_i, γ_i, ζ_i, Γ *and* B_i *heuristically;*
2. *set* D_i *to make* $\overline{D}_i \geq 0$ *in* (22)*;*
3. *select* Q *by condition* (4)*;*
4. *set* P *sufficiently large to guarantee both Equation (23):*

$$P \geq \frac{\rho(r^2 + Q)}{4\lambda_L} \max_{i=1,\cdots,N} \left(\frac{B_i + \sigma D_i}{\sigma B_i} \right), \tag{23}$$

and Equation (4) with:

$$\Delta = \frac{1}{2} \sum_{i=1}^{N} \frac{\lambda_{i2}}{B_i + \sigma D_i} \|\mathbf{s}_i(0)\|^2 + \frac{\overline{f}^2}{4\rho\Gamma};$$

5. *update* $K_i(t)$ *according to* (22) *to ensure that:*

$$\overline{K}_i(t) \geq \frac{1}{2}\rho\lambda_{i2}. \tag{24}$$

Proof. After substitution from (11), (21) leads to:

$$\begin{aligned}
\dot{V} \leq & - \sum_{i=1}^{N} \frac{\overline{K}_i(t)}{B_i + \sigma D_i} \mathbf{s}_i^{\mathsf{T}} \mathbf{s}_i - \sum_{i=1}^{N} \frac{\overline{D}_i}{B_i + \sigma D_i} \dot{\mathbf{x}}_i^{\mathsf{T}} \dot{\mathbf{x}}_i \\
& - \min_{i=1,\cdots,N} \left(\frac{\sigma B_i}{B_i + \sigma D_i} \right) \frac{4\lambda_L P}{r^2 + Q} V_p + \frac{\|\mathbf{f}\|^2}{4\Gamma} \\
\leq & - \frac{1}{2} \sum_{i=1}^{N} \frac{\rho\lambda_{i2}}{B_i + \sigma D_i} \mathbf{s}_i^{\mathsf{T}} \mathbf{s}_i - \rho V_p + \frac{\|\mathbf{f}\|^2}{4\Gamma} \\
\leq & - \rho V + \rho\chi\left(\|\mathbf{f}\|\right),
\end{aligned} \tag{25}$$

where $\overline{D}_i \geq 0$, (23)-(24) have been applied, and:

$$\chi(\|\mathbf{f}\|) = \frac{\|\mathbf{f}\|^2}{4\rho\Gamma}.$$

Time integration of $\dot{V}$ from 0 to $t \geq 0$ gives:

$$\begin{aligned}
V(t) \leq & e^{-\rho t} V(0) + \rho \int_0^t e^{-\rho(t-\tau)} \chi(\|\mathbf{f}(\tau)\|) d\tau \\
\leq & e^{-\rho t} V(0) + \rho \sup_{0 \leq \tau \leq t} \chi(\|\mathbf{f}(\tau)\|) \int_0^t e^{-\rho(t-\tau)} d\tau \\
\leq & e^{-\rho t} V(0) + \sup_{0 \leq \tau \leq t} \chi(\|\mathbf{f}(\tau)\|).
\end{aligned} \tag{26}$$

Then, Assumption 2 and (26) imply that:

$$V_p(t) \leq V(t) \leq e^{-\rho t} \cdot V_p(0) + \Delta \quad \forall t \geq 0,$$

and, with Proposition 2 and Q and P selected by (4), further imply that $\|\mathbf{x}_{ij}(t)\| < r \; \forall (i,j) \in \mathcal{E}(0)$, i.e., the connectivity of the teleoperated slave swarm is preserved. ■

The proof of objectives 1 and 2 of Problem 1, omitted due to page limit, shows input-to-state stable swarm [20].

Lemma 1 and Theorem 1 are the key contributions of this paper. The inequality (11), which holds for the tree network $\mathcal{G}(0)$, helps bound $\dot{V}$ in (21) and (25). Then, the time integration (26) and Proposition 2 imply that the distance between any slave pair $(i,j) \in \mathcal{E}(0)$ can be maintained strictly smaller than the communication radius r. Compared to autonomous MRS-s and leader-follower systems, the connectivity of a teleoperated swarm can be endangered by the unpredictable perturbation $\mathbf{f}$ sent from the master. Unlike external disturbances, e.g. wind forces, $\mathbf{f}$ conveys the operator command to the slave swarm and should be accepted unless it threatens swarm connectivity by moving the informed slave such that other slaves cannot keep up with it. The analysis in this section proves that the distributed controls (13) eliminate the threat posed by the operator command to connectivity if properly designed. Condition (24) on $K_i(t)$ exposes the uniqueness of the proposed design, especially in the P+d control form in Remark 1: it stiffens the inter-slave couplings and simultaneously increases their local damping injections on their relative distances. To the authors' best knowledge, the control (13) is among first to maintain the connectivity of a teleoperated swarm via a state-dependent updating law of the coupling and damping gains.

IV. Experiment Results

Figure 2 depicts the swarm teleoperation testbed used to validate the dynamic strategy (13). In it, a Novint Falcon device is the master and three Geomatic Touch robots are the slave swarm. All robots are connected to local computers that run the Robot Operating System (ROS) and MATLAB/Simulink. The positions of all slave end-effectors are controlled locally and, together with the inter-slave connections, are displayed on the master side in real-time for visual feedback to the operator.

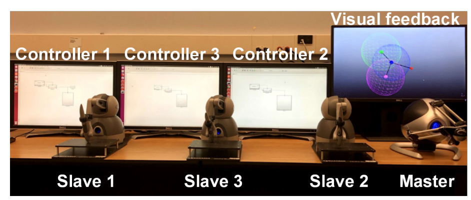


Fig. 2. The swarm teleoperation testbed used in the experimental validation.

Whereas the master-slave 1 connection is permanent, slave 1 exchanges positions with slaves 2 and 3 only when they are strictly closer than $r = 0.1$ m from each other. Hence, the initial tree network of the slave swarm is $\mathcal{G}(0) = \{\mathcal{V}, \mathcal{E}(0)\}$ with $\mathcal{V} = \{1, 2, 3\}$ and $\mathcal{E}(0) = \{(1,2), (1,3)\}$. The operator commands the motion of the slave swarm through saturated Proportional master-slave 1 control: $\mathbf{f} = \mathrm{Sat}\left(K \cdot (\mathbf{x}_0 - \mathbf{x}_1)\right)$, where $\mathrm{Sat}(\cdot)$ is the standard saturation and $\mathbf{x}_0$ is the position of the master end-effector. To avoid destabilizing the experiment, slave 1 is compliantly connected to the master robot with $K = 10$. The controls (13) aim to synchronize the slave swarm with the master while preserving interconnection links $(1,2)$ and $(1,3)$ under perturbation $\mathbf{f}$.

Figure 3 plots the task-space paths of all robot end-effectors. The operator first moves the master from A to B and to C slowly, and all slaves are synchronized tightly and follow the master. Then, the user moves the master from C to

D fast, increasing the perturbation $\mathbf{f}$ to the slave swarm. The paths of all slaves overshoot, but the slave swarm approaches the master. Because of gravity compensation errors of the swarm and compliant master-slave 1 connection $\mathbf{f}$, the slave swarm cannot track the master closely in the experiment.

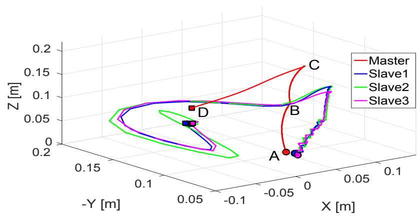


Fig. 3. Experimental task-space paths of all robot end-effectors.

Figure 4 plots the experimental inter-slave distances: 1-2 - the red line; 1-3 - the blue line. For small operator perturbation $\mathbf{f}$ (from A to C in Figure 3), the inter-slave position errors are smaller than 0.01 m. For increased perturbation $\mathbf{f}$ to the slave swarm(from C to D in Figure 3), those errors quickly grow to 0.03 m, at about the 27-th second. Yet, the controls (13) keep them strictly smaller than $r = 0.1$ m and preserve links $(1, 2)$ and $(1, 3)$ in the swarm tree network.

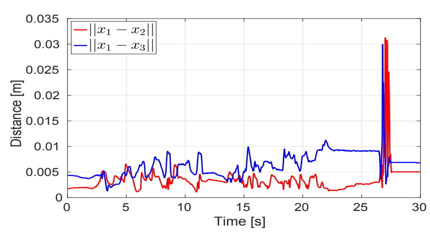


Fig. 4. Inter-slave distances: 1-2 - the red line; 1-3 - the blue line.

On the contrary, the experiment at `https://youtu.be/UDAJAbRszS0` shows that increased user perturbation $\mathbf{f}$ breaks the inter-slave links and thwarts swarm synchronization in the absence of the proposed algorithm.

V. Conclusions

The dynamic coupling and damping injection law in this paper preserves the tree network connectivity of a teleoperated swarm. After sliding surfaces reduce the order of the swarm dynamics, a customized potential function provides the dynamic couplings that, in combination with dynamic local dampings, prevent the dynamic mismatches induced by the sliding surfaces from changing the connectivity of the slave swarm. Rigorous energy analysis proves that all links of the tree network are preserved by the proposed dynamic regulation of the inter-slave couplings and of the local damping injections. Future work will consider connectivity-preserving swarm teleoperation with limited actuation and heterogeneous communication radius.

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