2021 IEEE International Conference on Robotics and Automation (ICRA 2021)
May 31 - June 4, 2021, Xi'an, China

Proportional and Reachable Cluster Teleoperation of a Distributed Multi-Robot System

Yuan Yang[1], Daniela Constantinescu[1] and Yang Shi[1]

2021 IEEE International Conference on Robotics and Automation (ICRA) | 978-1-7281-9077-8/21/$31.00 ©2021 IEEE | DOI: 10.1109/ICRA48506.2021.9561156

***Abstract*— A remote team of robots may be teleoperated by multiple users to explore unstructured environments and to tackle unforeseen emergencies therein. During a large-scale environmental search, each user may visually observe a unique hazard endangering the remote robot connected to their local robot. Therefore, each user may want to tele-drive the remote robot team to a location different than the target locations of other users. This paper resolves the possible conflicts among the multiple user commands through a distributed clustering algorithm that allocates to each user a number of remote robots proportional to the urgency of their request. A pivotal design challenge in the teleoperation context is to ensure that the remote robots allocated to each user are topologically reachable from the user's local robot within the induced communication subnetwork. The proposed design overcomes this challenge through a reachability-constrained integer linear program that modulates the interconnections of the remote robots on the fly. A comparative experiment on a platform with 2 local and 12 remote robots validates the practical efficacy of the proposed clustering algorithm.**

I. INTRODUCTION

A bilaterally teleoperated multi-robot system can leverage the unmatched perception and cognition of its human operators to successfully execute evolving and challenging tasks like search and rescue [1] in unknown and complex environments. However, it requires thoughtful design of the couplings among the multiple robots. Significant research addresses the design of the multi-robot interconnections for efficient coordination under physical constraints like obstacles [2], bearing-only measurements [3], connectivity maintenance [4] and object transportation [5]. Most recently, an extensive human subjects study [6] evaluates the control effort dedicated to connectivity maintenance in multi-robot teleoperation, and the teleoperation strategy [7] temporarily deletes and subsequently restores inter-robot connections during search in a cluttered environment.

Passive closed-loop multi-robot teleoperation systems are advantageous because they guarantee safe physical interactions between robots and users/environments [8]. They can be designed based on the Time-Domain Passivity Approach (TDPA). The TDPA provides a model-free teleoperation control framework amenable to integration with diverse techniques for cooperative landing of unmanned aerial vehicles on mobile platforms [9], bimanual object manipulation [10] and hierarchical pose regulation of aerial manipulators [11]. For enhanced transparency, the TDPA can be blended with tele-impedance [12], explicit force control [13] and a local virtual proxy [14].

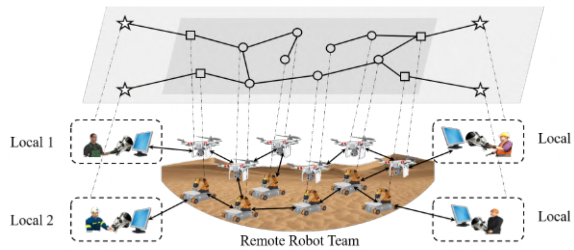


Fig. 1. A distributed multi-robot teleoperation system with $N_m = 4$ local robots and $N_s = 12$ remote robots.

The adaptive distribution of the control authority among the users can reduce their fatigue and improve task completion, and is achieved through several strategies. Specifically, a kinesthetic link between two users who control a shared remote robot determines the leader and the follower by measuring the internal energy transmissions [15]. A scalable TDPA-based mechanism enables a single trainer to select one active trainee in a multi-trainee haptic training system in real time [16]. A metric derived from Bayesian filtering adaptively adjusts the levels of robot autonomy and of human intervention in shared control [17]. It also informs the approach proposed in this paper for allocating remote robots to users during distributed multi-robot teleoperation. When tele-steering a shared network of remote robots, see Fig. 1, each user may want to guide a subset of the remote robots to reach/track their preferred target, distinct from the other users' targets, in response to changes in the environment. This paper formulates a proportional cluster teleoperation strategy that enables each user to tele-drive a number of remote robots proportional to the urgency of their request.

The intentions of the users and the situations they encounter evolve over time, changing the urgency of their requests throughout the teleoperation. Therefore, the robot network must be adaptively partitioned into multiple subgroups teleoperated by their corresponding users without mutual interference. An integer programming solution for multi-robot task allocation [18] provides the inspiration. Stimulated by promising applications in logistics [19] and patrolling [20], the latest variants of the integer program advance multi-robot task allocation by integrating path planning with stochastic costs [21] and obstacles [22], by mitigating the impact of reduced sensing quality [23], and by searching the optimum with the alternating direction method of multipliers [24]. They also unify the single-integrator dynamics with com-

[1]The authors are with the Department of Mechanical Engineering, University of Victoria, Victoria, BC V8W 2Y2 Canada (e-mail: yangyuan@uvic.ca; danielac@uvic.ca; yshi@uvic.ca).

munication constraints [25] and the control-affine dynamics with task priorities [26]. How to lend the integer program to the proportional cluster teleoperation of an Euler-Lagrange remote robot team remains an unanswered problem.

The main contribution of this paper is a distributed algorithm that adaptively divides a network of Euler-Lagrange remote robots into multiple subnetworks amenable to teleoperation by appropriate modulation of the inter-robot couplings. We assume that all remote robots can estimate the urgencies of all user requests with a consensus-based protocol that runs at a reasonably fast time scale. A key step in our design is to formulate the problem as a reachability-constrained integer program in which the remote robots detect their reachability from every local robot/human user within corresponding subnetworks. Then, an innovative coupling of a dedicated reachability detection mechanism with a saddle-point dynamical system forms a distributed algorithm that guides the remote robots to find their clusters. Finally, every pair of adjacent remote robots scales its coupling gain by the inner product of the decision variables of the pair, and the remote robots coupled to the users' local robots unilaterally weaken/restore their couplings to the local robots by their own decision variables. Thus, a subgroup of robots is allocated to, and can be effectively teleoperated by, a unique user. The size of the subgroup is proportional to the urgency of the user's request. A proof-of-concept experimental comparison validates the proposed algorithm and clarifies the role of the reachability constraint.

II. Problem Statement

Let a multi-robot teleoperation system like the one in Fig. 1 be deployed for large-scale search and rescue in a disaster environment. In this system: the group of N_s remote robots collectively explores the uncertain disaster environment; the N_m users manipulate their local robots from different places to tele-drive the remote robots while they monitor their exploration. Further, let each of the first N_m (leader) remote robots be interconnected respectively with a unique local robot, and the remaining $N_s - N_m \geq 0$ (follower) remote robots exchange information only with other remote robots over distributed communications.

When the leader remote robot $l \in \{1, \cdots, N_m\}$ encounters an emergency of a certain level, its associated user (user to whom it is connected) submits a corresponding bid to it through their local-remote communications, to gather other robots to its monitored area to execute a certain task collectively. Let the piecewise constant user bids $u_l(t)$ be selected from the set $\mathcal{U} = \{b_0, b_1, \cdots, b_m\} \subsetneq \mathbb{R}_{>0}$ by the users $l \in \{1, \cdots, N_m\}$. In response to the N_m user bids, the remote robot team is partitioned into N_m clusters $\mathcal{G}_l = \{\mathcal{V}_l, \mathcal{E}_l\}$, each teleoperated by one of the N_m users to handle its emergency simultaneously with, and independently of, all the other clusters handling their emergencies. When a leader remote robot l encounters a more urgent situation, its associated user l submits a higher bid u_l, and more remote robots are assigned to the cluster $\mathcal{G}_l$ for rapid task execution. Herein, we assume that all users are experts trained to objectively assess the levels of emergency and submit bids that match the encountered situations according to a unified standard. While future work will seek to mitigate the practical cognitive biases of users, herein we propose the following proportional cluster teleoperation control objective:

Definition 1. The number of remote robots allocated to the cluster $\mathcal{G}_l$ is proportional to the user bid u_l by

$$r_l = |\mathcal{V}_l| = \frac{N_s u_l}{u_1 + \cdots + u_{N_m}} \in \mathbb{Z}_{\geq 0}.$$

Note that we assume that N_m, N_s and $\mathcal{U}$ are such that all r_l-s are integers and $r_1 + \cdots + r_{N_m} = N_s$. This assumption may not always hold in practice. Therefore, how to relax Definition 1 to make every r_l an integer in the most general case remains an open problem for future study.

The remote robots in $\mathcal{G}_l$ should also be reachable from the local robot l, and minimize the sum of their topological distances to the local robot l in the communication network $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ of the remote robot team.

Definition 2. Given a cluster $\mathcal{G}_l = \{\mathcal{V}_l, \mathcal{E}_l\}$ of remote robots:

- a remote robot $i \in \mathcal{V}_l$ is topologically reachable from the local robot l if there exists a path between the remote robots i and l in $\mathcal{G}_l$;
- the cluster $\mathcal{G}_l$ is reachable from the local robot l if every remote robot in $\mathcal{G}_l$ is reachable from the local robot l;
- the clustering of the remote robot team is reachable if every cluster $\mathcal{G}_l$ is reachable from the associated local robot l.

In $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, each remote robot $i \in \mathcal{V} = \{1, \cdots, N_s\}$ is endowed with a vector of decision variables $\boldsymbol{\omega}_i = \left(\omega_i^1, \cdots, \omega_i^{N_m}\right)^\mathsf{T}$, where $\omega_i^l = 1$ if robot i is to be assigned to the cluster $\mathcal{G}_l$, and $\omega_i^l = 0$ otherwise, $\forall l = 1, \cdots, N_m$. Let each remote robot i be aware of its geodesic distance d_i^l to the leader remote robot l in $\mathcal{G}$, with $\mathcal{E}$ collecting all inter-robot communications. Then, the clustering of the remote robot team can be formulated as the following integer program:

$$\min_{\omega_i^l} \sum_{i=1}^{N_s} \sum_{l=1}^{N_m} d_i^l \omega_i^l, \tag{1a}$$

$$\text{s.t.} \sum_{l=1}^{N_m} \omega_i^l = 1,\ i = 1, \cdots, N_s, \tag{1b}$$

$$\sum_{i=1}^{N_s} \omega_i^l = r_l,\ l = 1, \cdots, N_m, \tag{1c}$$

$$\omega_i^l \in \{0, 1\},\ i = 1, \cdots, N_s,\ l = 1, \cdots, N_m, \tag{1d}$$

$$\mathcal{G}_l \text{ is reachable from the local robot } l = 1, \cdots, N_m. \tag{1e}$$

Here: the linear objective function in (1a) clusters the remote robots into $\mathcal{G}_l$, $l = 1, \cdots, N_m$ so as to minimize the sum of their geodesic distances in $\mathcal{G}$ to the leader remote robot l and to the local robot l; the constraint (1b) assigns every remote robot i to a unique cluster; the constraint (1c) groups r_l remote robots into the cluster $\mathcal{G}_l$; the constraint (1d) compels all decision variables ω_i^l to be either 0 or 1; and the constraint (1e) ensures that all the remote robots allocated to the cluster $\mathcal{G}_l$ are connected to and can thus be teleoperated by the associated local robot l.

Integer programming has been extensively leveraged for multi-robot task allocation, which is NP-hard due to the binary constraints on the optimization variables ω_i^l. In multi-robot teleoperation, however, the remote robot team to be partitioned is interconnected over a distributed communications network $\mathcal{G}$. Therefore, a reachability constraint (1e) must be placed on each cluster to ensure that every user l can teleoperate their cluster $\mathcal{G}_l$ of remote robots. The following example explains the unique challenge of clustering for teleoperation.

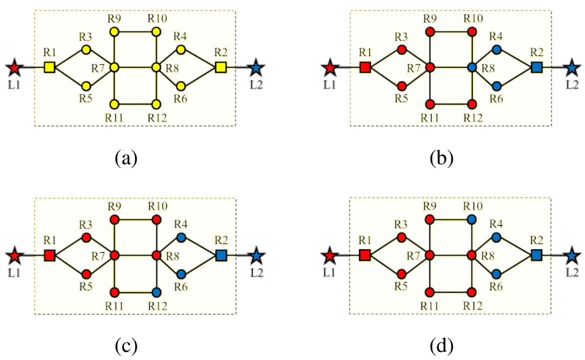


Fig. 2. Without the reachability constraint (1e), the integer program (1) has 3 solutions for the cluster teleoperation problem for a network of 2 local robots and 12 remote robots (Fig. 2(a)) under $r_1 = 8$ and $r_2 = 4$: the partition in Fig. 2(b) with both $\mathcal{G}_1$ and $\mathcal{G}_2$ connected is reachable; and the remaining two solutions in Fig. 2(c) and Fig. 2(d) are unreachable because $\mathcal{G}_2$ is disconnected.

Consider the multi-robot teleoperation system with the distributed communications shown in Fig. 2(a). For the user bids $u_1 = 2u_2$, Definition 1 requires to allocate $r_1 = 8$ and $r_2 = 4$ remote robots to two clusters $\mathcal{G}_1$ and $\mathcal{G}_2$ to be teleoperated by the local robots 1 and 2, respectively. User l can teleoperate the remote cluster $\mathcal{G}_l$ iff every remote robot in $\mathcal{G}_l$ is reachable from the local robot l. In the absence of the reachability constraint (1e), the integer program (1) has 3 minimizers $\boldsymbol{\omega}^*$: (i) $\omega_i^{1*} = \omega_j^{2*} = 1$ for $i = 1, 3, 5, 7, 9, 10, 11, 12$ and $j = 2, 4, 6, 8$, and $\omega_i^{1*} = \omega_j^{2*} = 0$ otherwise; (ii) $\omega_i^{1*} = \omega_j^{2*} = 1$ for $i = 1, 3, 5, 7, 8, 9, 10, 11$ and $j = 2, 4, 6, 12$, and $\omega_i^{1*} = \omega_j^{2*} = 0$ otherwise; and (iii) $\omega_i^{1*} = \omega_j^{2*} = 1$ for $i = 1, 3, 5, 7, 8, 9, 11, 12$ and $j = 2, 4, 6, 10$, and $\omega_i^{1*} = \omega_j^{2*} = 0$ otherwise. The three minimizers lead to the three possible clusterings depicted in Fig. 2(b)-Fig. 2(d). In these figures, the red and blue vertices indicate the remote robots assigned to the clusters $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively. Because the remote robots R12 in Fig. 2(c) and R10 in Fig. 2(d) are unreachable from the local robot L2, the clusterings enforced by the last two minimizers prevent user 2 from teleoperating the robot cluster $\mathcal{G}_2$. The novel reachability constraint (1e) precludes the pathological clusterings and preserves the reachable solution in Fig. 2(b).

III. Main Result

A. Handling the Constraints (1c) and (1e)

A distributed solution for (1) must properly account for two constraints: the constraint (1c), which demands that each remote robot infers the size r_l of each cluster $\mathcal{G}_l$; and the constraint (1e), which requires that each remote robot computes its reachability from the local robot l after joining the cluster $\mathcal{G}_l$. This section designs two continuous-time consensus-based algorithms to handle the two constraints in a distributed manner. Because the two algorithms converge exponentially in the analysis, their discrete-time equivalents converge linearly in the experiments.

Let every remote robot $i = 1, \cdots, N_s$ estimate the user bids u_l by $\widehat{u}_i^l$, $l = 1, \cdots, N_m$, with $\widehat{u}_i^l$ updated by the following consensus protocol:

$$\widehat{v}_i^l = \mathrm{Proj}_{\mathcal{U}}\big(\widehat{u}_i^l\big), \tag{2a}$$

$$\dot{\widehat{u}}_i^l = \begin{cases} \sum_{j\in\mathcal{N}_i} \big(\widehat{u}_j^l - \widehat{u}_i^l\big) + k_u\big(u_l - \widehat{u}_i^l\big), & \text{if } i = l, \\ \sum_{j\in\mathcal{N}_i} \big(\widehat{u}_j^l - \widehat{u}_i^l\big), & \text{if } i \neq l, \end{cases} \tag{2b}$$

where $\mathrm{Proj}_{\mathcal{U}}\big(\widehat{u}_i^l\big)$ projects $\widehat{u}_i^l$ onto the set $\mathcal{U}$ of candidate user bids. After concatenating the estimated user bids into $\widehat{\mathbf{u}}^l = \big(\widehat{u}_1^l, \cdots, \widehat{u}_{N_s}^l\big)^\top$ for $l = 1, \cdots, N_m$, the dynamics (2) can be grouped into

$$\dot{\widehat{\mathbf{u}}}^l = -\mathbf{L}\widehat{\mathbf{u}}^l - k_u\big(\widehat{u}_l^l - u_l\big)\mathbf{e}_l,$$

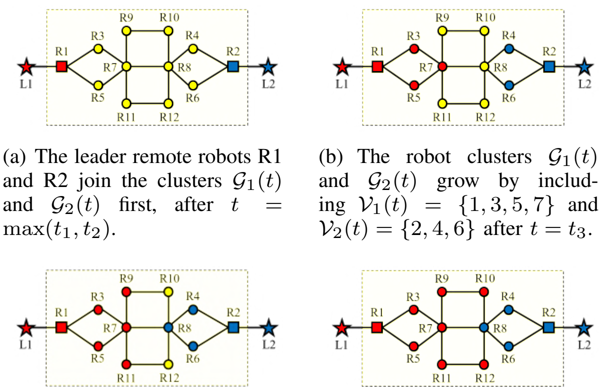
and further into

$$\dot{\widetilde{\mathbf{u}}}^l = -\mathbf{L}\big(\widetilde{\mathbf{u}}^l + k_u u_l \mathbf{1}\big) - k_u\mathbf{E}_l\widetilde{\mathbf{u}}^l = -\big(\mathbf{L} + k_u\mathbf{E}_l\big)\widetilde{\mathbf{u}}^l,$$

where $\mathbf{e}_l$ is the basis vector of $\mathbb{R}^{N_s}$ with the l-th element one and all other zero, $\widetilde{\mathbf{u}}^l = \widehat{\mathbf{u}}^l - u_l\mathbf{1}$ is the estimation error and $\mathbf{E}_l = \mathrm{diag}\{\mathbf{e}_l\}$. Along the error dynamics, the energy function $V_{ul} = \widetilde{\mathbf{u}}_l^\top\widetilde{\mathbf{u}}_l/2$ evolves with $\dot{V}_{ul} = -\widetilde{\mathbf{u}}^{l\top}\left(\mathbf{L} + k_u\mathbf{E}_l\right)\widetilde{\mathbf{u}}^l$. Because $\mathbf{L} + k_u\mathbf{E}_l$ is positive definite, $\widetilde{\mathbf{u}}^l \to \mathbf{0}$ exponentially, and $\widehat{u}_i^l$ and $\widehat{v}_i^l$ converge to u_l $\forall i = 1, \cdots, N_s$ and $\forall l = 1, \cdots, N_m$. Then, every remote robot i can adopt the output mapping $\widehat{r}_i^l = N_s\widehat{v}_i^l/\big(\widehat{v}_i^1 + \cdots + \widehat{v}_i^{N_m}\big)$ to calculate the corresponding size r_l of every remote robot cluster $\mathcal{G}_l$.

Definition 3. Given a remote robot cluster $\mathcal{G}_l$ with $\mathcal{V}_l \subsetneq \mathcal{V}$, let $\mathcal{N}_i^l \subseteq \mathcal{V}_l$ collect all remote robots $j \in \mathcal{V}_l$ that are adjacent to robot $i \notin \mathcal{V}_l$ in the remote robot network $\mathcal{G}$. Then, every remote robot $i \notin \mathcal{V}_l$ is adjacent to the cluster $\mathcal{G}_l$ if $\mathcal{N}_i^l \neq \emptyset$.

For every robot cluster $\mathcal{G}_l$, the decision variable is $\omega_i^l = 1$ if the remote robot i is grouped into the robot cluster $\mathcal{G}_l$, and $\omega_i^l = 0$ otherwise. Because only the leader remote robot l can communicate with the local robot l, the reachability constraint (1e) can be recast as: in every cluster $\mathcal{G}_l$, all remote robots are connected to the leader remote robot l. Let $\mathcal{G}_l(t)$ be the cluster $\mathcal{G}_l$ at time $t \geq 0$. Then, $\mathcal{G}_l(t)$ is reachable from the local robot l if $\mathcal{G}_l(t^-)$ is reachable from it and a new robot $i \notin \mathcal{V}_l(t^-)$ is coupled to any robot $j \in \mathcal{V}_l(t^-)$ at time t. Given that the remote robot cluster is initially empty, $\mathcal{G}_l(0) = \emptyset$, the reachability constraint (1e) ensures that the first robot added to the cluster $\mathcal{G}_l$ at some time $t > 0$ is the leader remote robot l. When the cluster $\mathcal{G}_l$ is no longer empty, only remote robots $i \notin \mathcal{V}_l$ adjacent to the cluster $\mathcal{G}_l$ can join $\mathcal{G}_l$ to maintain it reachable from the local robot l.



(a) The leader remote robots R1 and R2 join the clusters $\mathcal{G}_1(t)$ and $\mathcal{G}_2(t)$ first, after $t = \max(t_1, t_2)$.

(b) The robot clusters $\mathcal{G}_1(t)$ and $\mathcal{G}_2(t)$ grow by including $\mathcal{V}_1(t) = \{1, 3, 5, 7\}$ and $\mathcal{V}_2(t) = \{2, 4, 6\}$ after $t = t_3$.

(c) The robot R8 joins $\mathcal{G}_2(t)$ at $t = t_4$, and the robots R9 and R11 join $\mathcal{G}_1(t)$ at $t = t_5$.

(d) The robots R10 and R12 join $\mathcal{G}_1(t)$ last and complete the reachable clustering at $t = t_6$.

Fig. 3. An exemplary forming of two reachable clusters.

Fig. 3 illustrates step-by-step the reachability-constrained clustering of the example system in Fig. 2. Both clusters are initially empty and the leader remote robots R1 and R2 join $\mathcal{G}_1$ and $\mathcal{G}_2$ at times t_1 and t_2, respectively. During $t \in [0, t_l)$, no follower remote robot can join $\mathcal{G}_l$, $l = 1, 2$, because $\mathcal{N}_i^l(t) = \emptyset$, see Fig. 3(a). Afterwards, other robots that are adjacent to the clusters are assigned progressively. The decision-making dynamics may grow the clusters to $\mathcal{G}_1(t_3)$ with $\mathcal{V}_1(t_3) = \{1, 3, 5, 7\}$ and $\mathcal{G}_2(t_3)$ with $\mathcal{V}_2(t_3) = \{2, 4, 6\}$ at t_3. Then, the robot R8 is adjacent to both clusters, the robots R9 and R11 are adjacent only to $\mathcal{G}_1(t_3)$, and the robots R10 and R12 are not adjacent to any cluster at t_3, see Fig. 3(b). Because $r_1 = 8$ and $r_2 = 4$, the robot R8 then joins $\mathcal{G}_2$ at t_4, and the robots R9 and R11 join $\mathcal{G}_1$ at t_5, see Fig. 3(c). Now, the robots R10 and R12 become adjacent to both clusters and can join the cluster $\mathcal{G}_1$ at t_6, see Fig. 3(d). In doing so, they complete the formation of two reachable clusters $\mathcal{G}_l$ that the users l can teleoperate, $l = 1, 2$.

Let the remote robot i infer the reachability of cluster $\mathcal{G}_l$ by evaluating a local variable ζ_i^l, for $i = 1, \cdots, N_s$ and $l = 1, \cdots, N_m$. More specifically, $\zeta_i^l = 1$ if the robot i is adjacent to $\mathcal{G}_l$; and $\zeta_i^l = 0$ otherwise. Then, let ζ_i^l evolve by:

$$\zeta_i^l = \mathrm{Proj}_{[0,1]}\big(\gamma_i^l\big), \tag{3a}$$

$$\dot{\gamma}_i^l = \sum_{j \in \mathcal{N}_i} \omega_j^l \big(\gamma_j^l + \rho_i^l - \gamma_i^l - \rho_j^l\big) + k_\gamma \big(N_s e_i^l - \gamma_i^l\big), \tag{3b}$$

$$\dot{\rho}_i^l = \sum_{j \in \mathcal{N}_i} \omega_j^l \big(\gamma_j^l - \gamma_i^l\big), \tag{3c}$$

where $\mathrm{Proj}_{[0,1]}\big(\gamma_i^l\big)$ projects γ_i^l onto the interval $[0, 1]$, $k_\gamma > 0$ is a constant gain, and e_i^l is the l-th element of $\mathbf{e}_i$.

Let the cluster $\mathcal{G}_l$ be empty initially. Then, $\omega_j^l = 0$ for $j \in \mathcal{N}_i$ in (3), and $\dot{\gamma}_i^l = k_\gamma\big(N_s - \gamma_i^l\big)$ and $\dot{\rho}_i^l = 0$ for the leader remote robot $i = l$. Hence, the state γ_i^l and the output ζ_i^l converge exponentially to N_s and 1, respectively, while the state ρ_i^l remains invariant. For other robots $i \neq l$, the dynamics (3c) are $\dot{\gamma}_i^l = -k_\gamma \gamma_i^l$ and $\dot{\rho}_i^l = 0$. Their state γ_i^l and output ζ_i^l converge exponentially to 0, and ρ_i^l remains invariant. Thus, the leader remote robot l joins the cluster $\mathcal{G}_l$ if $r_l \geq 1$, while other remote robots stay outside $\mathcal{G}_l$.

After the leader remote robot l joins the cluster $\mathcal{G}_l$, $\omega_j^l = 1$ for $j \in \mathcal{N}_i^l$, and $\omega_j^l = 0$ for $j \notin \mathcal{N}_i^l$. For all robots i adjacent to $\mathcal{G}_l$, let $\boldsymbol{\gamma}_c^l$ and $\boldsymbol{\rho}_c^l$ stack γ_i^l and ρ_i^l, respectively. Then (3b) and (3c) become

$$\begin{aligned}\dot{\boldsymbol{\gamma}}_c^l &= -\big(\mathbf{L}_{cl} + k_\gamma \mathbf{I}\big)\boldsymbol{\gamma}_c^l + \mathbf{L}_{cl}\boldsymbol{\rho}_c^l + k_\gamma N_s \widehat{\mathbf{e}}_{cl}, \\ \dot{\boldsymbol{\rho}}_c^l &= -\mathbf{L}_{cl}\boldsymbol{\gamma}_c^l,\end{aligned}$$

where $\mathbf{L}_{cl}$ is the Laplacian matrix of $\mathcal{G}_l$, and $\widehat{\mathbf{e}}_{cl}$ has a single nonzero entry with value 1. Because $\mathcal{G}_l$ is connected, $\mathbf{L}_{cl}^\dagger \succeq \mathbf{0}$ and the generalized inverse of $\mathbf{L}_{cl}$ satifies $\mathbf{L}_{cl}\mathbf{L}_{cl}^\dagger = \mathbf{L}_{cl}^\dagger \mathbf{L}_{cl} = \mathbf{I} - \mathbf{1}\mathbf{1}^\mathsf{T}/|\mathcal{V}_l|$. Defining $\widetilde{\boldsymbol{\gamma}}_c^l = \boldsymbol{\gamma}_c^l - N_s \mathbf{1}/|\mathcal{V}_l|$ and $\widetilde{\boldsymbol{\rho}}_c^l = \boldsymbol{\rho}_c^l - k_\gamma \mathbf{L}_{cl}^\dagger\big(\mathbf{1}\mathbf{1}^\mathsf{T} - N_s\mathbf{I}\big)\widehat{\mathbf{e}}_{cl}$ leads to the error dynamics

$$\begin{aligned}\dot{\widetilde{\boldsymbol{\gamma}}}_c^l &= -\big(\mathbf{L}_{cl} + k_\gamma \mathbf{I}\big)\widetilde{\boldsymbol{\gamma}}_c^l + \mathbf{L}_{cl}\widetilde{\boldsymbol{\rho}}_c^l, \\ \dot{\widetilde{\boldsymbol{\rho}}}_c^l &= -\mathbf{L}_{cl}\widetilde{\boldsymbol{\gamma}}_c^l.\end{aligned}$$

Along these dynamics, the energy function $V_{\gamma\rho} = \widetilde{\boldsymbol{\gamma}}_c^{l\,\mathsf{T}}\widetilde{\boldsymbol{\gamma}}_c^l + \widetilde{\boldsymbol{\rho}}_c^{l\,\mathsf{T}}\widetilde{\boldsymbol{\rho}}_c^l$ varies according to $\dot{V}_{\gamma\rho} = -2\widetilde{\boldsymbol{\gamma}}_c^{l\,\mathsf{T}}\big(\mathbf{L}_{cl} + k_\gamma \mathbf{I}\big)\widetilde{\boldsymbol{\gamma}}_c^l$, and thus $\widetilde{\boldsymbol{\gamma}}_c^l \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\widetilde{\boldsymbol{\rho}}_c^l \in \mathcal{L}_\infty$. Then γ_i^l and ζ_i^l converge to $N_s/|\mathcal{V}_l| \geq 1$ and 1, respectively, while ρ_i^l remains bounded for every robot i that is adjacent to $\mathcal{G}_l$. If the remote robot i is not adjacent to $\mathcal{G}_l$, then $i \neq l$, and hence $\dot{\gamma}_i^l = -k_\gamma \gamma_i^l$ and $\dot{\rho}_i^l = 0$, and γ_i^l and ζ_i^l both converge exponentially to 0 while ρ_i^l is invariant. Thus, all robots $i \notin \mathcal{V}_l$ adjacent to $\mathcal{G}_l$ become candidate members of $\mathcal{G}_l$, but the robots i not adjacent to $\mathcal{G}_l$ cannot join the cluster $\mathcal{G}_l$.

B. Adapting the Inter-Robot Couplings

With the distributed protocols (2) estimating cluster sizes and (3) detecting reachability, the integer program (1) can be relaxed by

$$\min_{\boldsymbol{\omega}_i} \ \sum_{i=1}^{N_s} \mathbf{d}_i^\mathsf{T}\boldsymbol{\omega}_i, \tag{4a}$$

$$\text{s.t. } \mathbf{1}^\mathsf{T}\boldsymbol{\omega}_i = 1, \ i = 1, \cdots, N_s, \tag{4b}$$

$$\boldsymbol{\omega} + \big(\mathbf{B} \otimes \mathbf{I}_{N_m}\big)\boldsymbol{\upsilon} - \widehat{\mathbf{r}}/N_s = \mathbf{0}, \tag{4c}$$

$$\mathbf{0} \leq \boldsymbol{\omega}_i \leq \mathbf{1}, \ i = 1, \cdots, N_s, \tag{4d}$$

$$(\boldsymbol{\zeta}_i - \mathbf{1})^\mathsf{T}\boldsymbol{\omega}_i = 0, \ i = 1, \cdots, N_s, \tag{4e}$$

where, for each remote robot i, $\mathbf{d}_i = \big(d_i^1, \cdots, d_i^{N_m}\big)^\mathsf{T}$ stacks its geodesic distances to all leader remote robots $l = 1, \cdots, N_m$ in $\mathcal{G}$, and $\boldsymbol{\zeta}_i = \big(\zeta_i^1, \cdots, \zeta_i^{N_m}\big)^\mathsf{T}$ concatenates its reachability variables ζ_i^l. In (4c), $\boldsymbol{\omega} = \big(\boldsymbol{\omega}_1^\mathsf{T}, \cdots, \boldsymbol{\omega}_{N_s}^\mathsf{T}\big)^\mathsf{T}$, $\mathbf{B}$ is the incidence matrix of $\mathcal{G}$, the auxiliary variable $\boldsymbol{\upsilon} = \big(\boldsymbol{\upsilon}_1^\mathsf{T}, \cdots, \boldsymbol{\upsilon}_{|\mathcal{E}|}^\mathsf{T}\big)^\mathsf{T}$ facilitates the distributed search for the minimizer $\boldsymbol{\omega}^*$, and $\widehat{\mathbf{r}} = \big(\widehat{\mathbf{r}}_1^\mathsf{T}, \cdots, \widehat{\mathbf{r}}_{N_s}^\mathsf{T}\big)^\mathsf{T}$ stacks the estimated sizes $\widehat{\mathbf{r}}_i = \big(r_i^1, \cdots, r_i^{N_m}\big)^\mathsf{T}$ of the clusters.

A comparison of the relaxed program (4) to the integer program (1) shows that $\mathbf{d}_i^\mathsf{T}\boldsymbol{\omega}_i = \sum_{l=1}^{N_m} d_i^l \omega_i^l$ makes the objective functions (1a) and (4a) equivalent. Further,

$\mathbf{1}^{\mathsf{T}}\boldsymbol{\omega}_i = \sum_{l=1}^{N_m} \omega_i^l = 1$ converts (1b) into (4b). By (2), (4c) asymptotically approaches $\boldsymbol{\omega} + \big(\mathbf{B}\otimes\mathbf{I}_{N_m}\big)\boldsymbol{\upsilon} - \mathbf{1}\otimes\mathbf{r}/N_s = \mathbf{0}$ which, in turn, leads to $\mathbf{1}^{\mathsf{T}}\boldsymbol{\omega}^l = r_l$ with $\mathbf{r} = (r_1, \cdots, r_{N_m})^{\mathsf{T}}$ and $\boldsymbol{\omega}^l = \big(\omega_1^l, \cdots, \omega_{N_s}^l\big)^{\mathsf{T}}$. Given $\boldsymbol{\upsilon}$ unconstrained, (4c) can progressively approximate its counterpart (1c). Similar to designs for multi-robot task allocation, this paper convexifies the binary variable constraint (1d) by the box constraint (4d). Most importantly, the output $\mathbf{0} \le \boldsymbol{\zeta}_i \le \mathbf{1}$ of (3) together with (4d) rearrange (4e) into $\big(\zeta_i^l - 1\big)\omega_i^l = 0$ for $l = 1, \cdots, N_m$. If the remote robot i is not adjacent to $\mathcal{G}_l$, then $\zeta_i^l < 1$ and $\omega_i^l = 0$ by (4e). By (4e), robot i can join $\mathcal{G}_l$ with $\omega_i^l = 1$ only when it is adjacent to $\mathcal{G}_l$, i.e., $\zeta_i^l = 1$. Hence, the algorithm maintains the connectivity of every cluster $\mathcal{G}_l$. Moreover, all ζ_l^l converge to 1 for the leader remote robots first and at an exponential rate by (3). The objective function (4a) and $\big(\zeta_i^l - 1\big)\omega_i^l = 0$ dispatch $\omega_l^l = 1$ for the leader remote robots l above all. Together, (3) and (4e) guarantee the reachability constraint (1e).

The relaxed formulation (4) now permits the introduction of a distributed primal-dual algorithm for the remote robots to select their clusters. Define the Lagrangian function by

$$L = \sum_{i=1}^{N_s} \big[\mathbf{d}_i^{\mathsf{T}}\boldsymbol{\omega}_i + \lambda_i\big(\mathbf{1}^{\mathsf{T}}\boldsymbol{\omega}_i - 1\big) + \eta_i(\boldsymbol{\zeta}_i - \mathbf{1})^{\mathsf{T}}\boldsymbol{\omega}_i\big] + \boldsymbol{\mu}^{\mathsf{T}}\big[\boldsymbol{\omega} + (\mathbf{B}\otimes\mathbf{I}_{N_m})\boldsymbol{\upsilon} - \widehat{\mathbf{r}}/N_s\big],$$

where $i = 1, \cdots, N_s$, and λ_i, η_i and $\boldsymbol{\mu} = \big(\boldsymbol{\mu}_1^{\mathsf{T}}, \cdots, \boldsymbol{\mu}_{N_s}^{\mathsf{T}}\big)^{\mathsf{T}}$ with $\boldsymbol{\mu}_i = (\mu_i^1, \cdots, \mu_i^{N_m})^{\mathsf{T}}$ are dual variables. The saddle-point dynamics of L can search the minimizer of (4) by

$$k_\omega \dot{\boldsymbol{\omega}}_i = \mathrm{Proj}_{[\mathbf{0},\mathbf{1}]}\big[\boldsymbol{\omega}_i, -\mathbf{d}_i - \lambda_i\mathbf{1} - \kappa_i(\boldsymbol{\zeta}_i - \mathbf{1}) - \boldsymbol{\mu}_i\big], \quad (5a)$$

$$k_\lambda \dot{\lambda}_i = \mathbf{1}^{\mathsf{T}}\boldsymbol{\omega}_i - 1, \quad (5b)$$

$$k_\eta \dot{\eta}_i = (\boldsymbol{\zeta}_i - \mathbf{1})^{\mathsf{T}}\boldsymbol{\omega}_i, \quad (5c)$$

which each remote robot $i = 1, \cdots, N_s$, can execute locally, and by

$$k_\psi \dot{\boldsymbol{\upsilon}} = -\,(\mathbf{B}\otimes\mathbf{I}_{N_m})^{\mathsf{T}}\boldsymbol{\mu}, \quad (6a)$$

$$k_\mu \dot{\boldsymbol{\mu}} = \boldsymbol{\omega} + (\mathbf{B}\otimes\mathbf{I}_{N_m})\boldsymbol{\upsilon} - \widehat{\mathbf{r}}/N_s, \quad (6b)$$

where k_ω, k_λ, k_η, k_ψ and k_μ are positive constants. Let $\boldsymbol{\psi} = \big(\boldsymbol{\psi}_1^{\mathsf{T}}, \cdots, \boldsymbol{\psi}_{N_s}^{\mathsf{T}}\big)^{\mathsf{T}} = (\mathbf{B}\otimes\mathbf{I}_{N_m})\boldsymbol{\upsilon}$ and pre-multiply (6a) by $(\mathbf{B}\otimes\mathbf{I}_{N_m})$ to derive the following formulation of (6):

$$k_\psi \dot{\boldsymbol{\psi}}_i = -\sum_{j\in\mathcal{N}_i} \big(\boldsymbol{\mu}_i - \boldsymbol{\mu}_j\big), \quad (7a)$$

$$k_\mu \dot{\boldsymbol{\mu}}_i = \boldsymbol{\omega}_i + \boldsymbol{\psi}_i - \widehat{\mathbf{r}}_i/N_s, \quad (7b)$$

which admits a distributed implementation with 1-hop information exchanges. Jointly, (5) and(7) form a distributed clustering algorithm in which each remote robot i employs $\boldsymbol{\omega}_i$ to attenuate/restore their connections to neighbours in $\mathcal{G}$.

Let the subscripts li with $i = 1, \cdots, N_m$ and ri with $i = 1, \cdots, N_s$ index the local and remote robots, respectively, and assume that all robots have Euler-Lagrange dynamics:

$$\mathbf{M}_{li}(\mathbf{x}_{li})\ddot{\mathbf{x}}_{li} + \mathbf{C}_{li}(\mathbf{x}_{li}, \dot{\mathbf{x}}_{li})\dot{\mathbf{x}}_{li} = \mathbf{f}_{hi} + \mathbf{f}_{li}, \quad (8a)$$

$$\mathbf{M}_{ri}(\mathbf{x}_{ri})\ddot{\mathbf{x}}_{ri} + \mathbf{C}_{ri}(\mathbf{x}_{ri}, \dot{\mathbf{x}}_{ri})\dot{\mathbf{x}}_{ri} = \mathbf{f}_{ri}, \quad (8b)$$

where $\mathbf{x}_{*i}$ are the robot positions, $\mathbf{f}_{*i}$ are the control inputs, $* = l, r$, and $\mathbf{f}_{hi}$ are the forces exerted by the users.

To inform users of the remote robot clustering, the controls of the local robots $i = 1, \cdots, N_m$ render force feedback by

$$\mathbf{f}_{li} = P(\mathbf{x}_{ri} - \mathbf{x}_{li}) - D\dot{\mathbf{x}}_{li},$$

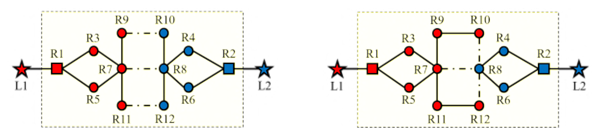
where P and D are positive constants. On the remote side, the leader remote robots $i = 1, \cdots, N_m$ are controlled by

$$\mathbf{f}_{ri} = \omega_i^i P(\mathbf{x}_{li} - \mathbf{x}_{ri}) + \sum_{j\in\mathcal{N}_i} \boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j P(\mathbf{x}_{rj} - \mathbf{x}_{ri}) - D\dot{\mathbf{x}}_{ri}$$

The follower remote robots $i = N_m + 1, \cdots, N_s$ cannot communicate with any local robots, and are controlled by

$$\mathbf{f}_{ri} = \sum_{j\in\mathcal{N}_i} \boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j P(\mathbf{x}_{rj} - \mathbf{x}_{ri}) - D\dot{\mathbf{x}}_{ri}.$$

Here: the proportional controls $P(\mathbf{x}_{ri} - \mathbf{x}_{li})$ in $\mathbf{f}_{li}$ with $i = 1, \cdots, N_m$ inform users about the distances between the local and the remote robots; $\omega_i^i P(\mathbf{x}_{li} - \mathbf{x}_{ri})$ in $\mathbf{f}_{ri}$ with $i = 1, \cdots, N_m$ allow the users i to tele-drive their assigned clusters with the leader remote robots included; the couplings among the remote robots are adapted by $\sum_{j\in\mathcal{N}_i} \boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j P(\mathbf{x}_{rj} - \mathbf{x}_{ri})$ in $\mathbf{f}_{ri}$ to synchronize only remote robots that belong to the same clusters; and $-D\dot{\mathbf{x}}_{li}$ and $-D\dot{\mathbf{x}}_{ri}$ inject damping to stabilize the system.



(a) $r_1 = r_2 = 6$. (b) $r_1 = 8$ and $r_2 = 4$.

Fig. 4. The interconnections between remote robots in the same clusters stay active (solid lines) while the interconnections between remote robots in different clusters become inactive (dash-dot lines).

The evolving coefficients $\boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j$ determine the status of the interconnections between the remote robots i and j. For $\boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j$ identical and positive constants, it can be shown that the elastic network of remote robots would be stretched to stay in the polyhedron spanned by all the local robots during teleoperation. In contrast, the distributed clustering dynamics (5) and (7) empower the remote robots i to select a cluster $\mathcal{G}_l$ by $\omega_i^l \to 1$ and to abandon other clusters $k \neq l$ by $\omega_i^k \to 0$. As a result, the coefficients $\boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j$ are updated by: $\boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j \to 1$ if the neighbouring robots i and j belong to the same cluster; and $\boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j \to 0$ if the neighbouring robots i and j belong to different clusters. In steady-state, every pair of neighbouring remote robots i and j in the same cluster activates their coupling with identical positive stiffness $\boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j P = P$, whereas each pair of neighbouring robots i and j in different clusters deactivates their coupling with vanishing stiffness $\boldsymbol{\omega}_i^{\mathsf{T}}\boldsymbol{\omega}_j P = 0$. Fig. 4 illustrates the strategy for activating/deactivating the interconnections among the remote robots for two exemplary cases.

The proportion estimator (2), reachability detector (3), clustering dynamics (5) and (7), and physical robot dynamics (8) form an interconnected bilateral teleoperation system whose rigorous passivity analysis remains an open topic for future research.

IV. Experimental Validation

This section demonstrates the effectiveness of the proposed distributed cluster teleoperation strategy experimentally. The experiments compare two cluster teleoperation algorithms derived from the integer program (1) with and without the reachability constraint (1e). The video of the experiments is available at https://youtu.be/ppabRiPBi6o[1].

Fig. 5. The experimental platform with 2 local robots and 12 remote robots connected across the communications network shown in Fig. 2(a).

In the experiments, two human users can teleoperate 12 remote robots using their 2 local robots, and can observe the end-effector positions of all robots on a screen, see Fig. 5. The users press buttons on their local robots to submit their bids $u_1, u_2 \in \mathcal{U} = \{1, 2\}$. The local and the leader remote robots are Geomagic Touch robots. All follower remote robots are Novint Falcon robots. Each robot is controlled locally via USB by a C++ program running on a dedicated Ubuntu machine at 1 kHz. The Robot Operating System handles all information exchanges among robots/machines.

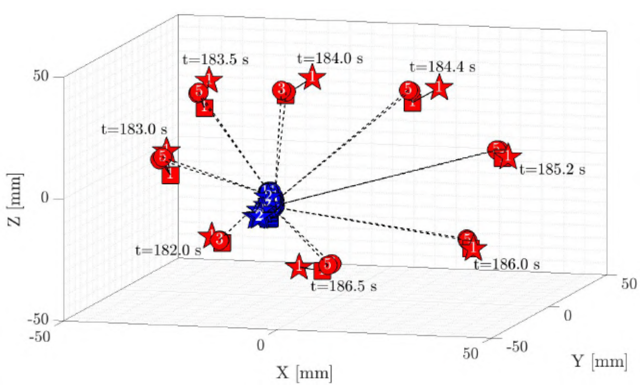


Fig. 6. The end-effector positions of all robots at different time instants during cluster teleoperation without the reachability constraint (1e). Given the user bids $u_1 = 1$ and $u_2 = 2$, the local robot L1 tele-drives 3 remote robots (R1, R3 and R5), and the local robot L2 holds the remaining 9 remote robots.

[1]The video is silent to avoid feeding into potential unconscious viewer biases.

Fig. 6 and Fig. 7 depict the end-effector positions of all robots at different time instants during cluster teleoperation, without the reachability constraint (1e) and with it, respectively. When the users submit the bids $u_1 = 1$ and $u_2 = 2$, Definition 1 prescribes that $r_1 = 4$ and $r_2 = 8$ remote robots must be distributed to the user 1 and to the user 2, respectively, for proportional cluster teleoperation. Without the reachability constraint (1e), the clustering algorithm derived from the integer program (1) assigns 3 remote robots (R1, R3 and R5) to $\mathcal{G}_1$, and 7 remote robots (R2, R4, R6, R7, R8, R10 and R12) to $\mathcal{G}_2$. Further, the convexification (4d) of (1d) leaves the remote robots R9 and R11 unassigned by $\boldsymbol{\omega}_9 = \boldsymbol{\omega}_{11} = (0.5, 0.5)^\mathsf{T}$. Thus, the designed control enables user 1 to tele-drive only the remote robots R1, R3 and R5, and permits user 2 to hold the remaining 9 robots, as shown in Fig. 6. In contrast, the clustering algorithm proposed in (5) and (7) considers the reachability constraint (1e) and achieves proportional cluster teleoperation. Note in Fig. 7 that the algorithm successfully allocates 4 remote robots (R1, R3, R5 and R7) to the local robot L1, and 8 remote robots (R2, R4, R6, R8, R9, R10, R11 and R12) to the local robot L2.

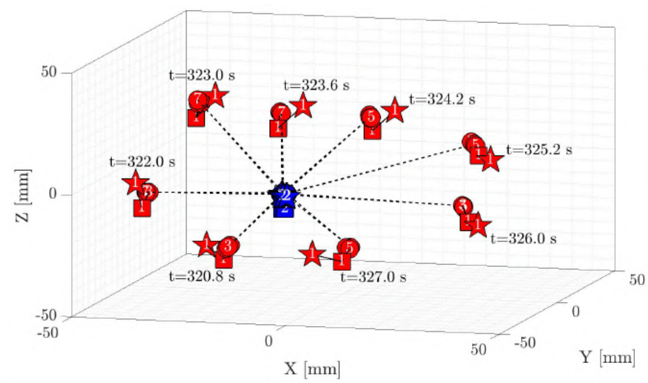


Fig. 7. The end-effector positions of all robots at different time instants during cluster teleoperation with the reachability constraint (1e). Given the user bids $u_1 = 1$ and $u_2 = 2$, the local robot L1 tele-drives 4 remote robots (R1, R3, R5 and R7), and the local robot L2 holds the remaining 8 remote robots.

V. Conclusion

This paper has formulated a proportional cluster teleoperation problem that it has approached through a reachability-constrained integer program. Two consensus-based protocols have enabled every remote robot to recognize the desired size of each upcoming cluster and to detect if they are reachable from each local robot in a distributed fashion. After relaxing the NP-hard integer program, the paper has devised a saddle-point algorithm to adapt the couplings between robots based on bids submitted by users during the teleoperation. An experimental comparison has illustrated the effectiveness of the proposed clustering teleoperation algorithm. Future work will develop a rigorous passivity analysis of the cluster teleoperation of a multi-robot system and will investigate its perceptual benefits through human user studies.

REFERENCES

[1] J. Casper and R. R. Murphy, "Human-robot interactions during the robot-assisted urban search and rescue response at the world trade center," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 33, no. 3, pp. 367–385, 2003.

[2] A. Franchi, C. Secchi, H. I. Son, H. H. Bülthoff, and P. R. Giordano, "Bilateral teleoperation of groups of mobile robots with time-varying topology," *IEEE Transactions on Robotics*, vol. 28, no. 5, pp. 1019–1033, 2012.

[3] A. Franchi, C. Masone, V. Grabe, M. Ryll, H. H. Bülthoff, and P. R. Giordano, "Modeling and control of UAV bearing formations with bilateral high-level steering," *The International Journal of Robotics Research*, vol. 31, no. 12, pp. 1504–1525, 2012.

[4] P. R. Giordano, A. Franchi, C. Secchi, and H. H. Bülthoff, "A passivity-based decentralized strategy for generalized connectivity maintenance," *The International Journal of Robotics Research*, vol. 32, no. 3, pp. 299–323, 2013.

[5] K. Y. Lui, H. Cho, C. Ha, and D. Lee, "First-person view semi-autonomous teleoperation of cooperative wheeled mobile robots with visuo-haptic feedback," *The International Journal of Robotics Research*, vol. 36, no. 5-7, pp. 840–860, 2017.

[6] M. Aggravi, C. Pacchierotti, and P. R. Giordano, "Connectivity-maintenance teleoperation of a UAV fleet with wearable haptic feedback," *IEEE Transactions on Automation Science and Engineering*, 2020, doi: 10.1109/TASE.2020.3000060.

[7] L. Sabattini, B. Capelli, C. Fantuzzi, and C. Secchi, "Teleoperation of multi-robot systems to relax topological constraints," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 4558–4564.

[8] F. Ferraguti, M. Bonfè, C. Fantuzzi, and C. Secchi, "Optimized power modulation in wave based bilateral teleoperation," *IEEE/ASME Transactions on Mechatronics*, 2020, doi: 10.1109/TMECH.2020.3013978.

[9] T. Muskardin, A. Coelho, E. R. D. Noce, A. Ollero, and K. Kondak, "Energy-based cooperative control for landing fixed-wing UAVs on mobile platforms under communication delays," *IEEE Robotics and Automation Letters*, vol. 5, no. 4, pp. 5081–5088, 2020.

[10] F. Porcini, D. Chiaradia, S. Marcheschi, M. Solazzi, and A. Frisoli, "Evaluation of an exoskeleton-based bimanual teleoperation architecture with independently passivated slave devices," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 10 205–10 211.

[11] A. Coelho, H. Singh, K. Kondak, and C. Ott, "Whole-body bilateral teleoperation of a redundant aerial manipulator," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 9150–9156.

[12] M. Laghi, A. Ajoudani, M. G. Catalano, and A. Bicchi, "Unifying bilateral teleoperation and tele-impedance for enhanced user experience," *The International Journal of Robotics Research*, vol. 39, no. 4, pp. 514–539, 2020.

[13] R. Balachandran, J. H. Ryu, M. Jorda, C. Ott, and A. Albu-Schäffer, "Closing the force loop to enhance transparency in time-delayed teleoperation," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 10 198–10 204.

[14] H. Singh, M. Panzirsch, A. Coelho, and C. Ott, "Proxy-based approach for position synchronization of delayed robot coupling without sacrificing performance," *IEEE Robotics and Automation Letters*, vol. 5, no. 4, pp. 6599–6606, 2020.

[15] N. A. Usmani, T. Kim, and J. Ryu, "Dynamic authority distribution for cooperative teleoperation," in *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2015, pp. 5222–5227.

[16] A. R. Licona, A. Lelevel, M. T. Pham, and D. Eberard, "A multi-trainee architecture for haptic hands-on training," in *2019 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2019, pp. 7314–7320.

[17] R. Balachandran, H. Mishra, M. Cappelli, B. Weber, C. Secchi, C. Ott, and A. Albu-Schäffer, "Adaptive authority allocation in shared control of robots using Bayesian filters," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 11 298–11 304.

[18] C. Nam and D. A. Shell, "Robots in the huddle: Upfront computation to reduce global communication at run time in multirobot task allocation," *IEEE Transactions on Robotics*, vol. 36, no. 1, pp. 125–141, 2020.

[19] S. Choudhury, K. Solovey, M. J. Kochenderfer, and M. Pavone, "Efficient large-scale multi-drone delivery using transit networks," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 4543–4550.

[20] C. D. Alvarenga, N. Basilico, and S. Carpin, "Multirobot patrolling against adaptive opponents with limited information," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 2486–2492.

[21] F. Yang and N. Chakraborty, "Chance constrained simultaneous path planning and task assignment for multiple robots with stochastic path costs," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 6661–6667.

[22] K. C. Tan, M. Jung, I. Shyu, C. Wan, and R. Dai, "Motion planning and task allocation for a jumping rover team," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 5278–5283.

[23] R. K. Ramachandran, N. Fronda, and G. S. Sukhatme, "Resilience in multi-robot target tracking through reconfiguration," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 4551–4557.

[24] R. N. Haksar, O. Shorinwa, P. Washington, and M. Schwager, "Consensus-based ADMM for task assignment in multi-robot teams," in *2019 International Symposium on Robotics Research (ISRR)*, 2019.

[25] B. Reily, C. Reardon, and H. Zhang, "Leading multi-agent teams to multiple goals while maintaining communication," in *Robotics: Science and Systems (RSS)*, 2020.

[26] Y. Emam, S. Mayya, G. Notomista, A. Bohannon, and M. Egerstedt, "Adaptive task allocation for heterogeneous multi-robot teams with evolving and unknown robot capabilities," in *2020 IEEE International Conference on Robotics and Automation (ICRA)*, 2020, pp. 7719–7725.