Robust Four-Channel Teleoperation Through Hybrid Damping-Stiffness Adjustment

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***Abstract*—This paper presents three strategies that adjust the coordination damping and stiffness of four-channel teleoperators to maintain the teleoperation stable regardless of time-varying delays in the transmission of operator and environment forces between the master and slave robots. A first strategy employs hybrid control terms that depend on position errors and local velocities. Thus, the hybrid terms simultaneously and dynamically regulate the master–slave coupling and the local damping injections. They increase the robustness of the system to perturbations caused by delayed transmission of operator and environment forces, but are singular at zero velocities. A second strategy injects additional damping around zero velocities, according to the master–slave position error. The additional damping makes the hybrid term nonsingular and eliminates chattering at zero velocities. However, it cannot synchronize the master and slave robots in the presence of large position error. This problem is addressed by a third strategy, which reduces the order of the position error in the hybrid term to guarantee the dominance of the Proportional term in coordination. Then, the two robots can be synchronized from arbitrarily large position errors. Lyapunov stability analysis and hardware-in-the-loop experimental results verify and compare these three proposed hybrid approaches.**

***Index Terms*—Bilateral teleoperation, force feedback, four-channel controller, position tracking, time-varying delays.**

I. Introduction

A TELEOPERATOR is a master–slave robotic system in which the motions of the master and slave robots are synchronized through the communication channel, with the goal of enabling a user to manipulate a remote environment [1]. When the teleoperator provides force feedback to the user in response to their motions, i.e., the teleoperation is bilateral [2], the mental load of the human operator can be reduced and their performance in assembly tasks in terms of task success rate and economy of exerted forces increases [3]. Therefore, bilateral teleoperation is typically used to enable users to manipulate hazardous or inaccessible environments, for example, outer space, subsea, and contaminated environments [4].

In addition to stability [5], bilateral teleoperators aim to provide precise position tracking and realistic force feedback, to closely couple the human operator to the remote environment [6], [7]. However, communication delays between the master and slave robots threaten the stability of the teleoperation and, implicitly, the safety of the robotic system and its human user [8]–[10]. Several control strategies have been developed to guarantee stable and high-fidelity teleoperation [2]. Among them, the passivity-based control is widely employed [11] because passivity of the teleoperator guarantees its stability in closed loop with passive operator and environment without explicit knowledge of the operator and environment models, using only teleoperator input and output measurements. Based on the analysis and design tools they use, three main passivity-based control approaches can be distinguished, scattering, damping injection, and adaptive strategies [11].

Scattering-based teleoperation, introduced in [8] and extended in [12] and [13] to address wave reflection, ensures passive communications in the presence of constant time delays by transmitting velocity and force information encoded as scattering variables between the master and slave robots. Modifications of classical scattering-based control have addressed time-varying delays [14] and position tracking [1]. Force tracking has been pursued through direct exchange of force information between the master and slave robots [6], [15]–[18].

Injection of variable or constant damping is another approach to guaranteeing passive time-delayed communications. Variable damping has typically been injected based on time-domain passivity arguments. In the time-domain passivity control approach, a time-domain passivity observer keeps track of the energy generated in the communications and a time-domain passivity controller injects the damping required to dissipate it [19]. The position drift problem has been mitigated through passivity-based feedback control [20]. An energy bounding approach for robust stabilization of teleoperation with time-varying delays has been presented in [21]. Sufficient conditions for passive communications with time-varying delays based on time-domain passivity arguments have been derived in [22]. The sudden activation of the controller at passivity breaches has been mitigated through power-based time-domain passivity control both for haptic interaction [23] and for multilateral teleoperation [24]. Transparent teleoperation with time-varying delays has been sought through four-channel time-domain passivity controllers [25]–[27]. Time-domain passivity has also been combined with scattering control in a nonlinear four-channel controller for position and force tracking [28].

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Constant damping has been injected mainly based on Lyapunov–Krasovskii energy analysis. A proportional-derivative plus local damping (PD+d) with gravity compensation controller for teleoperation with constant time delay has been introduced in [29]. Its rigorous stability proof has been presented in [30] where the simpler Proportional plus damping (P+d) controller has also been proposed. A sufficient condition for the stability of a flexible joint teleoperator with the PD+d controller has been obtained in [31]. In the absence of time delays, the need for velocity measurements has been eliminated through a first-order filter in [32]. A compensation component has been added to the PD+d controller in [33] to improve the interaction experience of the operator, but the added control terms cause wave reflections similar to those arising in scattering-based control. A position-force strategy with PD+d control at the slave and with force transmission to the master, with and without gravity compensation, has been offered in [34]–[36]. However, the slave control force rather than the environment force has been reflected to the master. The extension of the P + d controller to output feedback control has been reported in [32] and [37]–[39]. Actuator saturation has been considered through bounding the potentially destabilizing control terms in [39]–[41]. The compensation of the hand and environment forces at the master and slave sides during static contact has been studied in [42] and [43].

Adaptive passivity-based control strategies have been devised for various purposes in bilateral teleoperation with time-varying delays. Adaptation has been used to improve transparency [44], and to drive position errors and velocities to zero in the presence of parameter uncertainties [45], [46]. For constant inputs, it has served to track positions without gravity compensation [47]. For saturating actuators and lack of velocity measurements, it has been combined with a fast terminal sliding-mode velocity observer to guarantee asymptotic stability [41]. It has also been used for explicit compensation of the communication delay [48].

Recent experiments [49] have shown that tele-impedance [50] with force feedback is more robust than conventional four-channel teleoperation [5] for time-varying delays, but the four-channel controller constrains the master to the slave, and thus to the remote environment, tighter. The experiments in [49] indicate that the master–slave synchronization can be improved by designing four-channel teleoperation controllers that are robust to time-varying delays. To the best of authors' knowledge, the difficulty of guaranteeing passive force transmission across time-delayed communications has hindered four-channel teleoperation with static gains to date. In [51], a force-reflecting emulator control has been developed for a linear four-channel teleoperator with time-varying delays. However, extending and applying the approach to Euler–Lagrange (EL) teleoperation is not a trivial task because of the nonlinearity of EL dynamics. This paper is among the first to render a four-channel EL teleoperator robust to time-varying delays. It contributes three damping-stiffness adjustment strategies based on the integration of a hybrid control term into the conventional four-channel teleoperation architecture as follows.

1) A first strategy employs hybrid control terms dependent on position errors and local velocities. Thus, the hybrid terms simultaneously and dynamically regulate the master–slave coupling and the local damping injections. They increase robustness to perturbations caused by delayed transmission of operator and environment forces, but are singular at zero velocities.
2) A second strategy injects additional damping around zero velocities, according to the master–slave position error. The added damping eliminates the singularity of the hybrid term nonsingular and chattering at zero velocities at the expense of synchronization speed in the presence of large position errors.
3) A third strategy guarantees the dominance of the Proportional control in coordination by reducing the order of the position error in the hybrid term, thereby synchronizing the master and slave from arbitrarily large position errors.

All three strategies have a two-pronged action: they 1) modulate the master–slave coordination stiffness and 2) inject additional damping to dissipate the energy generated by the transmission of operator and environment forces across communications with time-varying delays. Thus, they make four-channel teleoperation robust to the destabilizing perturbations introduced in the feedback loop by the delayed transmission of forces between the master and slave robots [49]. Lyapunov–Krasovskii energy analysis leads to selection criteria for the control gains, and hardware-in-the-loop experiments validate the effectiveness of these three proposed strategies.

Notation is used as follows: bold upper $\mathbf{M}$ and lower fonts $\mathbf{a}$ indicate the matrices and vectors, respectively; $\mathbf{M}^\top$ and $\mathbf{M}^{-1}$ indicate the transpose and the inverse of $\mathbf{M}$; $\mathbf{I}$ is the unity matrix; $\|\mathbf{a}\|$ indicates the Euclidean norm of $\mathbf{a}$; $\mathcal{C}^0(\Lambda;\boldsymbol{\Omega})$, and $\mathcal{C}^1(\Lambda;\boldsymbol{\Omega})$, are the sets of continuous functions, and of functions with continuous derivative, defined on Λ with values in $\boldsymbol{\Omega}$; $T_r(t)\mathbf{x}$ is the "r-history" of $\mathbf{x}$ at time $t \in [a,b)$, i.e., $T_r(t)\mathbf{x} := \mathbf{x}(t+\theta)$ for $\theta \in [-r,0]$ and $\mathbf{x} : [a-r,b) \mapsto \mathbb{R}^n$ with $b > a > -\infty$ and $r > 0$; $\|\mathbf{x}\|_r := \max_{\theta\in[-r,0]} \|\mathbf{x}(\theta)\|$; a function ρ is positive definite if $\rho(0) = 0$ and $\rho(s) > 0$ for all $s \neq 0$; $\mathcal{K}^+$ is the set of positive definite continuous functions defined on $\mathbb{R}^+$; $\mathcal{K}$ is the set of positive definite, increasing and continuous functions; $\mathcal{K}_\infty$ is the set of positive definite, increasing and continuous functions with the property that $\lim_{s\to+\infty} \rho(s) = +\infty$; $\mathcal{KL}$ is the set of continuous functions $\beta(s,t) : \mathbb{R}^+ \times \mathbb{R}^+ \mapsto \mathbb{R}^+$ with the properties that: 1) for each $t \geq 0$, the mapping $\beta(\ldots,t)$ is of class $\mathcal{K}$ and 2) for each $s \geq 0$, the mapping $\beta(s,\cdot)$ is nonincreasing with $\lim_{t\to+\infty} \beta(s,t) = 0$. For simplicity of notation, the dependence on time of variables is not shown explicitly, for example, $\mathbf{q}$ replaces $\mathbf{q}(t)$.

II. Preliminaries

A. System Dynamics

Let the master and slave robots be n-degree-of-freedom serial manipulators with revolute joints and joint space dynamics

$$\begin{aligned}\mathbf{M}_m(\mathbf{q}_m)\ddot{\mathbf{q}}_m + \mathbf{C}_m(\mathbf{q}_m, \dot{\mathbf{q}}_m)\dot{\mathbf{q}}_m + \mathbf{g}_m(\mathbf{q}_m) &= \boldsymbol{\tau}_m + \boldsymbol{\tau}_h \\ \mathbf{M}_s(\mathbf{q}_s)\ddot{\mathbf{q}}_s + \mathbf{C}_s(\mathbf{q}_s, \dot{\mathbf{q}}_s)\dot{\mathbf{q}}_s + \mathbf{g}_s(\mathbf{q}_s) &= \boldsymbol{\tau}_s + \boldsymbol{\tau}_e \end{aligned} \quad (1)$$

where the index $i = m, s$ indicates the master and slave quantities, respectively, and for robot i: $\ddot{\mathbf{q}}_i$, $\dot{\mathbf{q}}_i$, and $\mathbf{q}_i$ are the joint acceleration, velocity, and position; $\mathbf{M}_i(\mathbf{q}_i)$ and $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ are the matrices of inertia and of Coriolis and centrifugal effects; $\mathbf{g}_i(\mathbf{q}_i)$ are the torques due to gravity; $\boldsymbol{\tau}_i$ are the control torques; and $\boldsymbol{\tau}_h$ and $\boldsymbol{\tau}_e$ are the user and environment torques.

The properties of the dynamics of robot i in (1), $i = m, s$, and the assumptions on time delays that facilitate the stability analysis in Section III are listed in the following.

- P.1 : The inertia matrix $\mathbf{M}_i(\mathbf{q}_i)$ is symmetric, positive definite, and uniformly bounded by $\mathbf{0} \prec \lambda_{i1}\mathbf{I} \preceq \mathbf{M}_i(\mathbf{q}_i) \preceq \lambda_{i2}\mathbf{I} \prec \infty$, with constants $\lambda_{i1} > 0, \lambda_{i2} > 0$.
- P.2 : The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew symmetric.
- A.1 : The time-varying time delays from robot i to robot j, d_i, are bounded, $0 \leq d_i \leq \overline{d}_i$, for $i, j = m, s$ and $i \neq j$.

Remark 1: Because the master and slave robots are n-link serial manipulators, their inertia matrices $\mathbf{M}_i(\mathbf{q}_i)$ are symmetric and positive definite [52]. Furthermore, it also guarantees the skew-symmetry property about $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ [52]. The two robots are assumed with revolute joints for ensuring that their inertia matrices can be uniformly bounded by $\mathbf{0} \prec \lambda_{i1}\mathbf{I} \leq \mathbf{M}_i(\mathbf{q}_i) \leq \lambda_{i2}\mathbf{I} \prec \boldsymbol{\infty}$ [52].

Remark 2: Assuming only upper bounds on the time-varying delays, bounded velocities of the master and slave robots and bounded position error between them have been guaranteed by P + d with gravity compensation control in [30], [34], and [53]. They have also verified its free motion synchronization performance experimentally. This paper extends the P + d with gravity compensation strategy to four channels with damping injection and gravity compensation control, to improve the master–slave synchronization in contact tasks.

B. Stability Notations

Let $U \subset \mathbb{R}^m$ be a nonempty set with $\mathbf{0} \in U$. Furthermore, let $\mathbf{x}(t)$ with $t \geq t_0$ be the unique solution of the initial-value problem

$$\begin{aligned}\dot{\mathbf{x}}(t) &= f(T_r(t)\mathbf{x}, \mathbf{u}(t)) \\ \mathbf{y}(t) &= h(t, \mathbf{x}(t)) \quad \mathbf{x}(t) \in \mathbb{R}^n, \ \mathbf{y}(t) \in \mathbb{R}^l, \ \mathbf{u}(t) \in U \end{aligned} \quad (2)$$

with initial condition $T_r(t_0)\mathbf{x} = \mathbf{x}_0 \in C^0([-r, 0]; \mathbb{R}^n)$, where: $r > 0$ is a constant; $T_r(t)\mathbf{x} := \mathbf{x}(t + \theta)$; $\theta \in [-r, 0]$; and the mappings $f : C^0([-r, 0]; \mathbb{R}^n) \times U \mapsto \mathbb{R}^n$ and $h : \mathbb{R}^+ \times \mathbb{R}^n \mapsto \mathbb{R}^l$ satisfy $f(\mathbf{0}, \mathbf{0}) = \mathbf{0}$ and $h(t, \mathbf{0}) = \mathbf{0}$ for all $t \in \mathbb{R}^+$.

D.1 [54]: System (2) is robustly forward complete (RFC) from input $\mathbf{u}(t)$ if

$$\sup_{0 \leq \xi \leq T} \|\mathbf{x}(t_0 + \xi)\| < +\infty, \quad \forall \|\mathbf{u}(t)\| \leq s, \forall \|\mathbf{x}_0\|_r \leq s$$

for every $s \geq 0$, $T \geq 0$ and $t_0 \in [0, T]$.

D.2 [54]: System (2) under hypotheses (S1-7) in [54] is input-to-output stable (IOS) with input $\mathbf{u}(t)$ and output $\mathbf{y}(t)$ if it is RFC from the input $\mathbf{u}(t)$ and there exist functions $\alpha(\cdot) \in \mathcal{K}$, $\beta(\cdot, \cdot) \in \mathcal{KL}$, such that the solution $\mathbf{x}(t)$ of (2) with $T_r(t_0)\mathbf{x} = \mathbf{x}_0$ satisfies

$$\|\mathbf{y}(t)\| \leq \max\left\{\beta(\|\mathbf{x}_0\|_r, t - t_0), \sup_{t_0 \leq \tau \leq t} \alpha(\|\mathbf{u}(\tau)\|)\right\} \quad (3)$$

for all $t \geq t_0$ and for all measurable $\mathbf{u}(t) \in U$, $(t_0, \mathbf{x}_0) \in \mathbb{R}^+ \times C^0([-r, 0]; \mathbb{R}^n)$.

D.3 [55]: System (2) under hypotheses (H1-4) in [55] with input $\mathbf{u}(t) \equiv \mathbf{0}$ is nonuniformly in time robustly globally asymptotically output stable (RGAOS) if it is RFC with the following properties.

a) For every $\epsilon > 0$ and $T \geq 0$, it holds that

$$\sup_{t_0 \leq t < +\infty} \|\mathbf{y}(t)\| < +\infty, \forall \|\mathbf{x}_0\|_r \leq \epsilon, \ t_0 \in [0, T].$$

b) For every $\epsilon > 0$ and $T \geq 0$, there exists $\delta(\epsilon, T) > 0$ such that

$$\|\mathbf{x}_0\|_r \leq \delta, \ t_0 \in [0, T] \Longrightarrow \|\mathbf{y}(t)\| \leq \epsilon, \quad \forall t \geq t_0.$$

c) For every $\epsilon > 0$, $T \geq 0$ and $R \geq 0$, there exists $\tau(\epsilon, T, R) \geq 0$ such that

$$\|\mathbf{x}_0\|_r \leq R, \ t_0 \in [0, T] \Longrightarrow \|\mathbf{y}(t)\| \leq \epsilon, \forall t \geq t_0 + \tau.$$

T.1 [54]: System (2) under hypotheses (S1-7) in [54] is IOS iff there exist: 1) a Lyapunov functional $V : \mathbb{R}^+ \times C^0([-r, 0]; \mathbb{R}^n) \mapsto \mathbb{R}^+$, which is almost Lipschitz on bounded sets; 2) functions α_1, α_2, α_3 of class $\mathcal{K}_\infty$; and 3) a locally Lipschitz positive definite function $\rho : \mathbb{R}^+ \mapsto \mathbb{R}^+$ such that

$$\alpha_1(\|h(t, \mathbf{x}(t))\|) \leq V(t, \mathbf{x}) \leq \alpha_2(\|\mathbf{x}\|_r) \quad (4)$$

for all $(t, \mathbf{x}) \in \mathbb{R}^+ \times C^0([-r, 0]; \mathbb{R}^n)$, and

$$\dot{V}(t, \mathbf{x}, f(\mathbf{x}, \mathbf{u})) \leq -\rho(V(t, \mathbf{x})) \quad (5)$$

for all $(\mathbf{x}, \mathbf{u}) \in C^0([-r, 0]; \mathbb{R}^n) \times U$ with $\alpha_3(\|\mathbf{u}\|) \leq V(t, \mathbf{x})$.

T.2 [54]: System (2) under hypotheses (S1-7) in [54] is IOS iff it is RFC from the input $\mathbf{u}(t)$ and is nonuniformly in time RGAOS with $\mathbf{u}(t) \equiv \mathbf{0}$.

Remark 3: The time-varying, nonlinear, and time-delayed teleoperation dynamics (1) are semiautonomous because they are forced by unpredictable user ($\boldsymbol{\tau}_h$) and environment ($\boldsymbol{\tau}_e$) perturbations. To date, robust position tracking control, especially P + d control [53], has offered a prominent control strategy for synchronizing the master and slave motions. However, the notion of IOS is relevant to systems which operate under external nonvanishing perturbations [54] and, thus, to teleoperation. After showing that the teleoperation dynamics (1) can be transformed into (2) by proper selection of state and input variables, this paper adopts the IOS definition D.2 for the robust stabilization of four-channel teleoperation with time-varying delays. Theorems T.1 and T.2, and definitions D.1 and D.3, will be used to prove IOS teleoperation.

The stability analysis in Section III relies on the following lemma.

L.1 [34]: For a positive-definite matrix $\boldsymbol{\Upsilon}$ and arbitrary vectors $\mathbf{a}(t)$ and $\mathbf{b}(\xi)$ with appropriate dimensions, the following inequality holds:

$$\pm 2\mathbf{a}^{\mathsf{T}}(t)\int_{t-d(t)}^{t}\mathbf{b}(\xi)d\xi - \int_{t-d(t)}^{t}\mathbf{b}^{\mathsf{T}}(\xi)\boldsymbol{\Upsilon}\mathbf{b}(\xi)d\xi \le \overline{d}\mathbf{a}^{\mathsf{T}}(t)\Upsilon^{-1}\mathbf{a}(t), \quad \forall \mathbf{a}(t), \mathbf{b}(t), 0 \le d(t) \le \overline{d}.$$

III. Controller Design and Stability Analysis

Four-channel teleoperation is optimal for master–slave coordination and transparent interaction [5], but is not robust to the delayed exchange of operator and environment forces between the master and slave robots [49]. This section proposes the three damping-stiffness adjustment control strategies that stabilize four-channel teleoperation across communications with time-varying delays. A first strategy introduces a hybrid term mixing position error and local velocity, which enables a hybrid damping-stiffness gain adjustment that provably dissipates the energy injected in the feedback loop by the transmission with time-varying delays of operator and environment forces. Then, a nonsingular version overcomes chattering by eliminating the torque spikes of the first hybrid control at zero velocities. A third strategy reduces the order of the position error in the hybrid term to guarantee: 1) master–slave coordination for arbitrarily large position errors and 2) tight coupling between the two robots.

A. Hybrid Damping-Stiffness Adjustment

Let nonlinear position- and velocity-dependent control terms be added to conventional P + d with gravity compensation teleoperation control [53], together with direct transmission of hand and environment torques

$$\begin{aligned}\boldsymbol{\tau}_m &= -\dot{\mathbf{q}}_m^{\star}(\mathbf{q}_m - \mathbf{q}_{sd})^{\mathsf{T}}\mathbf{B}_m(\mathbf{q}_m - \mathbf{q}_{sd}) - \mathbf{K}_m\dot{\mathbf{q}}_m \\ &\quad -\mathbf{P}(\mathbf{q}_m - \mathbf{q}_{sd}) + \mathbf{g}_m + \boldsymbol{\tau}_{ed} \\ \boldsymbol{\tau}_s &= -\dot{\mathbf{q}}_s^{\star}(\mathbf{q}_s - \mathbf{q}_{md})^{\mathsf{T}}\mathbf{B}_s(\mathbf{q}_s - \mathbf{q}_{md}) - \mathbf{K}_s\dot{\mathbf{q}}_s \\ &\quad -\mathbf{P}(\mathbf{q}_s - \mathbf{q}_{md}) + \mathbf{g}_s + \boldsymbol{\tau}_{hd}. \end{aligned} \tag{6}$$

Here, $\mathbf{q}_{id} = \mathbf{q}_i(t - d_i(t))$, $i = m, s$, and $\boldsymbol{\tau}_{hd} = \boldsymbol{\tau}_h(t - d_m(t))$ and $\boldsymbol{\tau}_{ed} = \boldsymbol{\tau}_e(t - d_s(t))$, where $d_i(t)$ is the time delay in the communication channel from robot i to the other robot; $\mathbf{B}_i$, $\mathbf{K}_i$, and $\mathbf{P}$ are the positive diagonal gain matrices; and $\dot{\mathbf{q}}_i^{\star} = [\dot{q}_{i1}^{\star}, \ldots, \dot{q}_{in}^{\star}]^{\mathsf{T}}$ is a vector with the property

$$\dot{q}_{ik}^{\star} = \begin{cases}\dfrac{1}{l\dot{q}_{ik}} & \dot{q}_{ik} \ne 0 \\ 0 & \dot{q}_{ik} = 0\end{cases} \Longrightarrow \dot{\mathbf{q}}_i^{\mathsf{T}}\dot{\mathbf{q}}_i^{\star} = \begin{cases}1 & \dot{\mathbf{q}}_i \ne 0 \\ 0 & \dot{\mathbf{q}}_i = 0\end{cases}$$

where $k = 1, \ldots, n$ and l is the number of joints with nonzero velocity.

In (2), the time-varying delays introduce distortions $-\mathbf{P}(\mathbf{q}_s - \mathbf{q}_{sd})$ and $-\mathbf{P}(\mathbf{q}_m - \mathbf{q}_{md})$ in the Proportional control terms $-\mathbf{P}(\mathbf{q}_m - \mathbf{q}_{sd})$ and $-\mathbf{P}(\mathbf{q}_s - \mathbf{q}_{md})$, respectively, and make the passivity of the transmitted torques $\boldsymbol{\tau}_{hd}$ an $\boldsymbol{\tau}_{ed}$ uncertain, leading to energy accumulation in the closed-loop teleoperator and threatening its stability and safety. Damping is conventionally injected to dissipate, or suitably limit, the harmful energy. The hybrid strategy in (2) injects damping via a control term nonlinear in the velocity of the local robot and the position error between the master and slave. The hybrid term has a twofold action: it adjusts the amount of locally injected damping and it regulates the stiffness of the master–slave coupling. Because $-\dot{\mathbf{q}}_i^{\star}(\mathbf{q}_i - \mathbf{q}_{jd})^{\mathsf{T}}\mathbf{B}_i(\mathbf{q}_i - \mathbf{q}_{jd})$, $i,\ j = m, s$ and $i \ne j$, has components opposite to those of $\dot{\mathbf{q}}_i$, the hybrid term injects additional damping to robot i, with gain dependent on the position error between the master and slave. On the other hand, the nonlinear term strengthens (weakens) the master–slave coupling when robot i's velocity is opposite to (aligned with) the Proportional control $-\mathbf{P}(\mathbf{q}_i - \mathbf{q}_{jd})$. This is because the hybrid term acts simultaneously as a Proportional control with matrix gain $-\dot{\mathbf{q}}_i^{\star}(\mathbf{q}_i - \mathbf{q}_{jd})^{\mathsf{T}}\mathbf{B}_i$ nonlinear in velocity. This behavior of the proposed hybrid control term is advantageous because adding damping increases energy dissipation and enhancing the master–slave coupling increases the stability robustness of closed-loop teleoperation.

As shown in [49], the external torques $\boldsymbol{\tau}_e$ and $\boldsymbol{\tau}_h$ due to environment and operator forces help coordinate the two robots both in free motion and in contact, but their delayed versions $\boldsymbol{\tau}_{ed}$ and $\boldsymbol{\tau}_{hd}$ include undesirable components that damage the coupling between the robots and can even lead to finite-time escaping velocities. Distinguishing the desirable from the harmful components and rejecting the latter is not trivial. In P + d control [53], the time-varying delays induce distotions $-\mathbf{P}(\mathbf{q}_j - \mathbf{q}_{jd})$ only in the local Proportional control term $-\mathbf{P}(\mathbf{q}_i - \mathbf{q}_{jd})$, $i,\ j = m, s$ and $i \ne j$, and with magnitude dependent on the velocity of the remote robot, i.e., $-\mathbf{P}(\mathbf{q}_j - \mathbf{q}_{jd}) = -\mathbf{P}\int_{t-d_j}^{t}\dot{\mathbf{q}}_j(\sigma)d\sigma$. Suitable design can ensure that fixed local damping serves double duty: it bounds the control disturbances at the remote robot by bounding the velocity of the local robot; and it rejects the local bounded control disturbances. In four-channel teleoperation, the time-varying delays distort not only the Proportional control term but also the transmission of external torques. Then, a fixed damping sufficient to reject the torque disturbances cannot be determined because the distortions contained and buried in external forces and their transmission are not known *a priori*. The proposed hybrid damping-stiffness adjustment makes four-channel teleoperation robust to delay-induced distortions with no need of identifying them. Because the nonlinear hybrid terms switch at zero velocities, the control torques are not smooth but chattering and noisy. A nonsingular hybrid strategy that eliminates chattering and lessens the control noise follows in Section III-B.

The stability of the system (1) under the control (6) is investigated using the following Lyapunov–Krasovskii functional:

$$V = V_k + V_p + V_d \tag{7}$$

with

$$V_k = \frac{1}{2}\dot{\mathbf{q}}_m^\top \mathbf{M}_m \dot{\mathbf{q}}_m + \frac{1}{2}\dot{\mathbf{q}}_s^\top \mathbf{M}_s \dot{\mathbf{q}}_s$$
$$V_p = \frac{1}{2}(\mathbf{q}_m - \mathbf{q}_s)^\top \mathbf{P}(\mathbf{q}_m - \mathbf{q}_s)$$
$$V_d = \sum_{i=m,s}\int_{-\overline{d}_i}^{0}\int_{t+\theta}^{t} e^{-\gamma\cdot(t-\xi)}\cdot \dot{\mathbf{q}}_i^\top(\xi)\mathbf{Q}_i\dot{\mathbf{q}}_i(\xi)d\xi d\theta$$

where V_k is the kinetic energy of the master and slave robots; V_p is the potential energy in the Proportional control; and V_d measures the energy injected by the time-varying delays in the forward and backward communication channels. The exponential decay coefficient $e^{-\gamma\cdot(t-\xi)}$ facilitates the construction of a term $-\gamma\cdot V_d$ needed in the stability proof and guarantees $V_d \geq 0$ for positive definite diagonal matrices $\mathbf{Q}_i$, $i = m, s$.

Lemma 1: Define the state and the input of the closed-loop teleoperation system to be $\mathbf{x} = [\dot{\mathbf{q}}_m^\top\ \dot{\mathbf{q}}_s^\top\ (\mathbf{q}_m - \mathbf{q}_s)^\top]^\top$ and $\mathbf{u} = [(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed})^\top\ (\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd})^\top]^\top$, respectively. The derivative of the Lyapunov–Krasovskii functional (7) can be upper bounded by

$$\dot{V} \leq -\gamma\cdot V - \mathbf{x}^\top \boldsymbol{\Psi}\mathbf{x} + \chi(\|\mathbf{u}\|) \tag{8}$$

where the diagonal blocks of the symmetric matrix $\boldsymbol{\Psi} = \text{diag}\{\boldsymbol{\Psi}_{11}, \boldsymbol{\Psi}_{22}, \boldsymbol{\Psi}_{33}\}$ are

$$\boldsymbol{\Psi}_{11} = \mathbf{K}_m - \frac{\upsilon_m + \gamma\lambda_{m2}}{2}\cdot\mathbf{I} - \overline{d}_m\mathbf{Q}_m - \frac{\overline{d}_s e^{\gamma\overline{d}_s}}{4\eta_s}\cdot\mathbf{P}\mathbf{Q}_s^{-1}\mathbf{P}^\top$$
$$\boldsymbol{\Psi}_{22} = \mathbf{K}_s - \frac{\upsilon_s + \gamma\lambda_{s2}}{2}\cdot\mathbf{I} - \overline{d}_s\mathbf{Q}_s - \frac{\overline{d}_m e^{\gamma\overline{d}_m}}{4\eta_m}\cdot\mathbf{P}\mathbf{Q}_m^{-1}\mathbf{P}^\top$$
$$\boldsymbol{\Psi}_{33} = \mathbf{B}_m - \frac{\overline{d}_s e^{\gamma\overline{d}_s}}{1-\eta_s}\cdot\mathbf{B}_m\mathbf{Q}_s^{-1}\mathbf{B}_m^\top + \mathbf{B}_s - \frac{\overline{d}_m e^{\gamma\overline{d}_m}}{1-\eta_m}\cdot\mathbf{B}_s\mathbf{Q}_m^{-1}\mathbf{B}_s^\top - \frac{\gamma}{2}\cdot\mathbf{P} \tag{9}$$

with $0 < \eta_i < 1$, $\upsilon_i > 0$, $i = m, s$, $\upsilon = \min(\upsilon_m, \upsilon_s)$ and

$$\chi(\|\mathbf{u}\|) = \frac{1}{2\upsilon}\cdot\|\mathbf{u}\|^2.$$

Proof: Using property P.2, the derivative of V_k is computed as

$$\begin{aligned}
\dot{V}_k = &-\dot{\mathbf{q}}_m^\top\mathbf{K}_m\dot{\mathbf{q}}_m - (\mathbf{q}_m - \mathbf{q}_{sd})^\top\mathbf{B}_m(\mathbf{q}_m - \mathbf{q}_{sd})\\
&-\dot{\mathbf{q}}_m^\top\mathbf{P}(\mathbf{q}_m - \mathbf{q}_{sd}) + \dot{\mathbf{q}}_m^\top(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed})\\
&-\dot{\mathbf{q}}_s^\top\mathbf{K}_s\dot{\mathbf{q}}_s - (\mathbf{q}_s - \mathbf{q}_{md})^\top\mathbf{B}_s(\mathbf{q}_s - \mathbf{q}_{md})\\
&-\dot{\mathbf{q}}_s^\top\mathbf{P}(\mathbf{q}_s - \mathbf{q}_{md}) + \dot{\mathbf{q}}_s^\top(\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd})\\
\leq &-\dot{\mathbf{q}}_m^\top\mathbf{K}_m\dot{\mathbf{q}}_m - \dot{\mathbf{q}}_m^\top\mathbf{P}(\mathbf{q}_m - \mathbf{q}_s) + \dot{\mathbf{q}}_m^\top(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed})\\
&-(\mathbf{q}_m - \mathbf{q}_s)^\top\mathbf{B}_m(\mathbf{q}_m - \mathbf{q}_s) - \dot{\mathbf{q}}_m^\top\mathbf{P}\int_{t-d_s}^{t}\dot{\mathbf{q}}_s(\xi)d\xi\\
&-2(\mathbf{q}_m - \mathbf{q}_s)^\top\mathbf{B}_m\int_{t-d_s}^{t}\dot{\mathbf{q}}_s(\xi)d\xi\\
&-\dot{\mathbf{q}}_s^\top\mathbf{K}_s\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_s^\top\mathbf{P}(\mathbf{q}_s - \mathbf{q}_m) + \dot{\mathbf{q}}_s^\top(\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd})\\
&-(\mathbf{q}_m - \mathbf{q}_s)^\top\mathbf{B}_s(\mathbf{q}_m - \mathbf{q}_s) - \dot{\mathbf{q}}_s^\top\mathbf{P}\int_{t-d_m}^{t}\dot{\mathbf{q}}_m(\xi)d\xi\\
&-2(\mathbf{q}_s - \mathbf{q}_m)^\top\mathbf{B}_s\int_{t-d_m}^{t}\dot{\mathbf{q}}_m(\xi)d\xi.
\end{aligned} \tag{10}$$

Without loss of generality, (10) assumes that $\dot{\mathbf{q}}_m$ and $\dot{\mathbf{q}}_s$ are not zero. Otherwise, $\dot{\mathbf{q}}_m^\top(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) = 0$ and/or $\dot{\mathbf{q}}_s^\top(\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd}) = 0$, respectively. The terms coupling velocities and external torques can be upper bounded by

$$\dot{\mathbf{q}}_m^\top(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) + \dot{\mathbf{q}}_s^\top(\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd}) \leq \sum_{i=m,s}\frac{\upsilon_i}{2}\cdot\dot{\mathbf{q}}_i^\top\dot{\mathbf{q}}_i + \frac{\|\mathbf{u}\|^2}{2\upsilon}. \tag{11}$$

The time derivative of V_p is

$$\dot{V}_p = (\mathbf{q}_m - \mathbf{q}_s)^\top\mathbf{P}\dot{\mathbf{q}}_m - (\mathbf{q}_m - \mathbf{q}_s)^\top\mathbf{P}\dot{\mathbf{q}}_s \tag{12}$$

and algebraic manipulation of V_d leads to

$$V_d = \sum_{i=m,s} e^{-\gamma t}\int_{-\overline{d}_i}^{0}\int_{t+\theta}^{t} e^{\gamma\xi}\cdot\dot{\mathbf{q}}_i^\top(\xi)\mathbf{Q}_i\dot{\mathbf{q}}_i(\xi)d\xi d\theta$$

whose derivative can be bounded as follows:

$$\begin{aligned}
\dot{V}_d = &-\sum_{i=m,s}\gamma\cdot e^{-\gamma t}\int_{-\overline{d}_i}^{0}\int_{t+\theta}^{t} e^{\gamma\xi}\cdot\dot{\mathbf{q}}_i^\top(\xi)\mathbf{Q}_i\dot{\mathbf{q}}_i(\xi)d\xi d\theta\\
&+\sum_{i=m,s} e^{-\gamma t}\int_{-\overline{d}_i}^{0}\big[e^{\gamma t}\cdot\dot{\mathbf{q}}_i^\top(t)\mathbf{Q}_i\dot{\mathbf{q}}_i(t)\\
&\qquad - e^{\gamma(t+\theta)}\cdot\dot{\mathbf{q}}_i^\top(t+\theta)\mathbf{Q}_i\dot{\mathbf{q}}_i(t+\theta)\big]d\theta\\
= &-\gamma\cdot V_d + \sum_{i=m,s}\overline{d}_i\dot{\mathbf{q}}_i^\top\mathbf{Q}_i\dot{\mathbf{q}}_i\\
&-\sum_{i=m,s} e^{-\gamma t}\int_{-\overline{d}_i}^{0} e^{\gamma(t+\theta)}\cdot\dot{\mathbf{q}}_i^\top(t+\theta)\mathbf{Q}_i\dot{\mathbf{q}}_i(t+\theta)d\theta\\
= &-\gamma\cdot V_d + \sum_{i=m,s}\overline{d}_i\dot{\mathbf{q}}_i^\top\mathbf{Q}_i\dot{\mathbf{q}}_i\\
&-\sum_{i=m,s} e^{-\gamma t}\int_{t-\overline{d}_i}^{t} e^{\gamma\xi}\cdot\dot{\mathbf{q}}_i^\top(\xi)\mathbf{Q}_i\dot{\mathbf{q}}_i(\xi)d\xi\\
\leq &\sum_{i=m,s}\overline{d}_i\dot{\mathbf{q}}_i^\top\mathbf{Q}_i\dot{\mathbf{q}}_i - e^{-\gamma\overline{d}_i}\int_{t-\overline{d}_i}^{t}\dot{\mathbf{q}}_i^\top(\xi)\mathbf{Q}_i\dot{\mathbf{q}}_i(\xi)d\xi - \gamma\cdot V_d\\
\leq &\sum_{i=m,s}\overline{d}_i\dot{\mathbf{q}}_i^\top\mathbf{Q}_i\dot{\mathbf{q}}_i - e^{-\gamma\overline{d}_i}\int_{t-d_i}^{t}\dot{\mathbf{q}}_i^\top(\xi)\mathbf{Q}_i\dot{\mathbf{q}}_i(\xi)d\xi - \gamma\cdot V_d.
\end{aligned} \tag{13}$$

Then, Lemma L.1 in [34] leads to

$$-\dot{\mathbf{q}}_m^\top\mathbf{P}\int_{t-d_s}^{t}\dot{\mathbf{q}}_s(\xi)d\xi - \eta_s\cdot e^{-\gamma\overline{d}_s}\int_{t-d_s}^{t}\dot{\mathbf{q}}_s^\top(\xi)\mathbf{Q}_s\dot{\mathbf{q}}_s(\xi)d\xi \leq \frac{\overline{d}_s e^{\gamma\overline{d}_s}}{4\eta_s}\cdot\dot{\mathbf{q}}_m^\top\mathbf{P}\mathbf{Q}_s^{-1}\mathbf{P}^\top\dot{\mathbf{q}}_m$$

for $\mathbf{a}(t) = (1/2)\mathbf{P}^\top\dot{\mathbf{q}}_m$, $\mathbf{b}(\xi) = \dot{\mathbf{q}}_s(\xi)$, and $\boldsymbol{\Upsilon} = \eta_s\cdot e^{-\gamma\overline{d}_s}\mathbf{Q}_s$, and to

$$\begin{aligned}
-2(\mathbf{q}_m - \mathbf{q}_s)^\top\mathbf{B}_m&\int_{t-d_s}^{t}\dot{\mathbf{q}}_s(\xi)d\xi - (1-\eta_s)\\
&\cdot e^{-\gamma\overline{d}_s}\int_{t-d_s}^{t}\dot{\mathbf{q}}_s^\top(\xi)\mathbf{Q}_s\dot{\mathbf{q}}_s(\xi)d\xi\\
&\leq \frac{\overline{d}_s e^{\gamma\overline{d}_s}}{1-\eta_s}\cdot(\mathbf{q}_m - \mathbf{q}_s)^\top\mathbf{B}_m\mathbf{Q}_s^{-1}\mathbf{B}_m^\top(\mathbf{q}_m - \mathbf{q}_s)
\end{aligned}$$

for $\mathbf{a}(t) = \mathbf{B}_m^\top(\mathbf{q}_m - \mathbf{q}_s)$, $\mathbf{b}(\xi) = \dot{\mathbf{q}}_s(\xi)$, and $\boldsymbol{\Upsilon} = (1-\eta_s)\cdot e^{-\gamma \overline{d}_s}\mathbf{Q}_s$. Adding the above-mentioned two inequalities leads to

$$\begin{aligned}-\big[\dot{\mathbf{q}}_m^\top\mathbf{P} + 2(\mathbf{q}_m - \mathbf{q}_s)^\top\mathbf{B}_m\big]\int_{t-d_s}^{t}\dot{\mathbf{q}}_s(\xi)d\xi \\ -e^{-\gamma\overline{d}_s}\int_{t-d_s}^{t}\dot{\mathbf{q}}_s^\top(\xi)\mathbf{Q}_s\dot{\mathbf{q}}_s(\xi)d\xi \\ \le \frac{\overline{d}_s e^{\gamma\overline{d}_s}}{1-\eta_s}\cdot(\mathbf{q}_m-\mathbf{q}_s)^\top\mathbf{B}_m\mathbf{Q}_s^{-1}\mathbf{B}_m^\top(\mathbf{q}_m-\mathbf{q}_s) \\ +\frac{\overline{d}_s e^{\gamma\overline{d}_s}}{4\eta_s}\cdot\dot{\mathbf{q}}_m^\top\mathbf{P}\mathbf{Q}_s^{-1}\mathbf{P}^\top\dot{\mathbf{q}}_m\end{aligned}\tag{14}$$

and, similarly, to

$$\begin{aligned}-\big[\dot{\mathbf{q}}_s^\top\mathbf{P} + 2(\mathbf{q}_s - \mathbf{q}_m)^\top\mathbf{B}_s\big]\int_{t-d_m}^{t}\dot{\mathbf{q}}_m(\xi)d\xi \\ -e^{-\gamma\overline{d}_m}\int_{t-d_m}^{t}\dot{\mathbf{q}}_m^\top(\xi)\mathbf{Q}_m\dot{\mathbf{q}}_m(\xi)d\xi \\ \le \frac{\overline{d}_m e^{\gamma\overline{d}_m}}{1-\eta_m}\cdot(\mathbf{q}_m-\mathbf{q}_s)^\top\mathbf{B}_s\mathbf{Q}_m^{-1}\mathbf{B}_s^\top(\mathbf{q}_m-\mathbf{q}_s) \\ +\frac{\overline{d}_m e^{\gamma\overline{d}_m}}{4\eta_m}\cdot\dot{\mathbf{q}}_s^\top\mathbf{P}\mathbf{Q}_m^{-1}\mathbf{P}^\top\dot{\mathbf{q}}_s.\end{aligned}\tag{15}$$

Property P.1 permits to bound the sum of V_k and V_p by

$$V_k + V_p \le \sum_{i=m,s}\frac{\lambda_{i2}}{2}\cdot\dot{\mathbf{q}}_i^\top\dot{\mathbf{q}}_i + \frac{1}{2}\cdot(\mathbf{q}_m-\mathbf{q}_s)^\top\mathbf{P}(\mathbf{q}_m-\mathbf{q}_s).\tag{16}$$

Adding (10), (12), and (13) and using the inequalities (11), (14), (15), and (16) lead to (8). □

Remark 4: A bilateral teleoperation system is typically semiautonomous because the human operator and environment are involved in the closed-loop system. However, modeling the two external terminators is not a trivial task since they are time-varying and unpredictable. Instead, their actions (forces) are external excitations for the master–slave subsystem. Then, the control policy is to synchronize the master and slave robots under the user and environment perturbations. Conventional four-channel teleoperation can tightly constrain the master and slave robots but becomes unstable in the presence of delayed force transmissions [49]. This paper aims to make four-channel teleoperation robust to time-varying delays. To this end: 1) it regards the sum of the user and transmitted environment torques $(\boldsymbol{\tau}_h+\boldsymbol{\tau}_{ed})$, and the sum of the environment and transmitted user torques $(\boldsymbol{\tau}_e+\boldsymbol{\tau}_{hd})$, as input $\mathbf{u}$ to the master, and to the slave, robot, respectively, and 2) it selects the velocities $(\dot{\mathbf{q}}_i, i=m,s)$ of, and the position error $(\mathbf{q}_m-\mathbf{q}_s)$ between, the master and slave as the state variables to control to achieve robust position tracking performance.

By Lemma 1, the following theorem serves as design criterion for the controller (6) for teleoperation systems with asymmetric time-varying delays.

Theorem 1: The teleoperation system (1) in closed loop with the controller (6) is IOS with input $\mathbf{u}(t) = [(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed})^\top\ (\boldsymbol{\tau}_e+\boldsymbol{\tau}_{hd})^\top]^\top$, state $\mathbf{x}(t) = [\dot{\mathbf{q}}_m^\top\ \dot{\mathbf{q}}_s^\top\ (\mathbf{q}_m-\mathbf{q}_s)^\top]^\top$, and output $\mathbf{y}(t) = h(t,\mathbf{x}(t)) = \mathbf{x}(t)$ if there exist positive definite matrix control gains $\mathbf{K}_i$, $\mathbf{B}_i$, $\mathbf{P}$, and positive scalars γ, η_i and υ_i, $i=m,s$, such that the diagonal blocks of matrix $\boldsymbol{\Psi}$ in (9) are positive semidefinite matrices.

Proof: After selecting the input $\mathbf{u}(t) = [(\boldsymbol{\tau}_h+\boldsymbol{\tau}_{ed})^\top\ (\boldsymbol{\tau}_e+\boldsymbol{\tau}_{hd})^\top]^\top$, state $\mathbf{x}(t) = [\dot{\mathbf{q}}_m^\top\ \dot{\mathbf{q}}_s^\top\ (\mathbf{q}_m-\mathbf{q}_s)^\top]^\top$, and output $\mathbf{y}(t) = \mathbf{x}(t)$, the teleoperation dynamics (1) can be transformed into the state-space equation (2) in [54, Theorem T.1] with $r = \max(\overline{d}_m, \overline{d}_s)$. Furthermore, V selected in (7) is almost Lipschitz on bounded sets. The selection

$$\alpha_1(\|h(t,\mathbf{x}(t))\|) = \frac{1}{2}\cdot\min(\lambda_{m1},\lambda_{s1},\underline{P})\cdot\|h(t,\mathbf{x}(t))\|^2$$

where $\underline{P}$ is the smallest eigenvalue of $\mathbf{P}$, makes $\alpha_1(\cdot)$ of class $\mathcal{K}_\infty$, and

$$V \ge \sum_{i=m,s}\frac{\lambda_{i1}}{2}\cdot\|\dot{\mathbf{q}}_i\|^2 + \frac{\underline{P}}{2}\cdot\|\mathbf{q}_m-\mathbf{q}_s\|^2 \ge \alpha_1(\|h(t,\mathbf{x}(t))\|).$$

From $\dot{\mathbf{q}}_i^\top(\xi)\mathbf{Q}_i\dot{\mathbf{q}}_i(\xi) \le \overline{Q}_i\cdot\|\dot{\mathbf{q}}_i(\xi)\|^2 \le \overline{Q}_i\cdot\|\mathbf{x}(\xi)\|^2$, it follows that:

$$\begin{aligned}\int_{-\overline{d}_i}^{0}\int_{t+\theta}^{t}\overline{Q}_i\cdot\|\mathbf{x}(\xi)\|^2 d\xi d\theta &= \int_{-\overline{d}_i}^{0}\int_{\theta}^{0}\overline{Q}_i\cdot\|\mathbf{x}(t+\tau)\|^2 d\tau d\theta \\ &\le \int_{-\overline{d}_i}^{0}\int_{\theta}^{0}\overline{Q}_i\cdot\|\mathbf{x}\|_r^2 d\xi d\theta \\ &= \frac{1}{2}\cdot\overline{d}_i^2\cdot\overline{Q}_i\cdot\|\mathbf{x}\|_r^2\end{aligned}$$

and thus

$$V \le \frac{1}{2}\sum_{i=m,s}\left(\lambda_{i2}\cdot\|\dot{\mathbf{q}}_i\|^2 + \overline{d}_i^2\cdot\overline{Q}_i\cdot\|\mathbf{x}\|_r^2\right) + \frac{\overline{P}}{2}\cdot\|\mathbf{q}_m-\mathbf{q}_s\|^2 \le \alpha_2(\|\mathbf{x}\|_r)$$

where

$$\alpha_2(\|\mathbf{x}\|_r) = \frac{1}{2}\cdot\left[\max(\lambda_{m2},\lambda_{s2},\overline{P}) + \overline{d}_m^2\overline{Q}_m + \overline{d}_s^2\overline{Q}_s\right]\cdot\|\mathbf{x}\|_r^2$$

is of class $\mathcal{K}_\infty$, and $\overline{P}$ and $\overline{Q}_i$ are the largest eigenvalues of $\mathbf{P}$ and $\mathbf{Q}_i$, respectively. Thus, the condition (4) of Theorem T.1 in [54] is satisfied.

Making $\boldsymbol{\Psi}$ positive semidefinite simplifies (8) by

$$\dot{V} \le -\gamma\cdot V + \chi(\|\mathbf{u}(t)\|)\tag{17}$$

for any state $\mathbf{x}$. After choosing $\alpha_3(\|\mathbf{u}\|) = (2/\gamma)\cdot\chi(\|\mathbf{u}(t)\|)$ of class $\mathcal{K}_\infty$, and $\rho(V) = (\gamma/2)\cdot V$ of class $\mathcal{K}$, (17) implies that the second condition (5) of theorem T.1 in [54] is guaranteed. Then, the closed-loop teleoperation system is IOS with the selected input, state, and output. □

Remark 5: The velocities of, and position error between, the master and slave of an IOS four-channel teleoperation system are bounded if the user and environment forces are bounded. In particular, $\mathcal{L}_\infty$ stability, already adopted for robust position tracking [53], ensures that the master and slave robots are coupled. This property implies that the operator can command the slave to desired locations by operating the master. In the absence of user and environment perturbations,

$\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$, IOS teleoperation synchronizes the master and slave robots by definition D.2. When the slave is in static contact with the environment, the user and environment forces are constant and balanced, and thus, $\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed} = \boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd} = \mathbf{0}$. Then, IOS four-channel teleoperation would eliminate the master–slave position error. Hardware-in-the-loop experiments in Section IV will illustrate the tight position coordination between the master and the slave and, thus, between the user and the environment, during contact.

B. Nonsingular Hybrid Damping-Stiffness Adjustment

When a joint velocity approaches zero in the presence of nonzero position error in (6), the nonlinear hybrid term opposing the joint velocity overwhelms the Proportional control along the respective joint space direction and stops the motion of the joint. When the joint velocity becomes zero, the nonlinear term becomes zero and Proportional control drives the respective joint again. The control (6) then leads to spikes in the control torque and to chatter in the joint velocity. To eliminate this behavior, this section proposes the following nonsingular hybrid damping-stiffness adjustment strategy:

$$\begin{aligned}\boldsymbol{\tau}_m &= -\mathbf{S}_m(\dot{\mathbf{q}}_m)\cdot(\mathbf{q}_m - \mathbf{q}_{sd})^{\mathsf{T}}\mathbf{B}_m(\mathbf{q}_m - \mathbf{q}_{sd}) - \mathbf{K}_m\dot{\mathbf{q}}_m\\ &\quad -\mathbf{P}(\mathbf{q}_m - \mathbf{q}_{sd}) + \mathbf{g}_m + \boldsymbol{\tau}_{ed}\\ \boldsymbol{\tau}_s &= -\mathbf{S}_s(\dot{\mathbf{q}}_s)\cdot(\mathbf{q}_s - \mathbf{q}_{md})^{\mathsf{T}}\mathbf{B}_s(\mathbf{q}_s - \mathbf{q}_{md}) - \mathbf{K}_s\dot{\mathbf{q}}_s\\ &\quad -\mathbf{P}(\mathbf{q}_s - \mathbf{q}_{md}) + \mathbf{g}_s + \boldsymbol{\tau}_{hd}\end{aligned} \tag{18}$$

where

$$\mathbf{S}_i(\dot{\mathbf{q}}_i) = \left[\frac{\mathrm{sat}(\dot{q}_{i1})\cdot\mathrm{sgn}(\dot{q}_{i1})}{n\dot{q}_{i1}}, \ldots, \frac{\mathrm{sat}(\dot{q}_{in})\cdot\mathrm{sgn}(\dot{q}_{in})}{n\dot{q}_{in}}\right]^{\mathsf{T}}$$

$$\mathrm{sat}(\dot{q}_{ik}) = \begin{cases}\dot{q}_{ik} & |\dot{q}_{ik}| \le \gamma/2n\\ \bar{\dot{q}}_i\cdot\mathrm{sgn}(\dot{q}_{ik}) & |\dot{q}_{ik}| > \gamma/2n\end{cases}$$

with $\gamma > 0$, $\mathrm{sgn}(\cdot)$ the signum function, $i = m, s$, and $k = 1, \ldots, n$ indexing the master and slave joints. The hybrid strategy in (18) replaces the singular terms $\dot{\mathbf{q}}_m^\star$ and $\dot{\mathbf{q}}_s^\star$ in (6) with the nonsingular terms $\mathbf{S}_m(\dot{\mathbf{q}}_m)$ and $\mathbf{S}_s(\dot{\mathbf{q}}_s)$. When a joint velocity $\dot{q}_{ik}$ approaches zero in the presence of nonzero position error, $\mathrm{sat}(\dot{q}_{ik})\cdot\mathrm{sgn}(\dot{q}_{ik})/n\dot{q}_{ik} = \mathrm{sgn}(\dot{q}_{ik})/n$ and the control spikes and joint chatter are eliminated.

The stability of the system (1) with the nonsingular hybrid damping-stiffness adjustment control (18) is investigated using the same Lyapunov–Krasovskii functional as in (3).

Lemma 2: Let $\sigma = \sigma_m + \sigma_s$ with $\sigma_i = \sum_{k=1}^n |\mathrm{sat}(\dot{q}_{ik})|$, $i = m, s$. Define the same state $\mathbf{x}$ and input $\mathbf{u}$ as in Lemma 1. The derivative of the Lyapunov–Krasovskii functional (7) can be upper bounded by

$$\dot{V} \le -\sigma\cdot(V_k + V_p) - \gamma\cdot V_d - \mathbf{x}^{\mathsf{T}}\boldsymbol{\Psi}\mathbf{x} + \sigma\cdot\chi(\|\mathbf{u}\|) \tag{19}$$

where $\boldsymbol{\Psi} = \mathrm{diag}\{\boldsymbol{\Psi}_{11}, \boldsymbol{\Psi}_{22}, \boldsymbol{\Psi}_{33}\}$ with

$$\boldsymbol{\Psi}_{11} = \mathbf{K}_m - \overline{d}_m\mathbf{Q}_m - \frac{\overline{d}_s e^{\gamma\overline{d}_s}}{4\eta_s}\cdot\mathbf{P}\mathbf{Q}_s^{-1}\mathbf{P}^{\mathsf{T}} - \omega_m\cdot\mathbf{I}$$

$$\boldsymbol{\Psi}_{22} = \mathbf{K}_s - \overline{d}_s\mathbf{Q}_s - \frac{\overline{d}_m e^{\gamma\overline{d}_m}}{4\eta_m}\cdot\mathbf{P}\mathbf{Q}_m^{-1}\mathbf{P}^{\mathsf{T}} - \omega_s\cdot\mathbf{I}$$

$$\begin{aligned}\boldsymbol{\Psi}_{33} &= \sigma_m\mathbf{B}_m - \frac{\overline{d}_s\sigma_m^2 e^{\gamma\overline{d}_s}}{1-\eta_s}\cdot\mathbf{B}_m\mathbf{Q}_s^{-1}\mathbf{B}_m^{\mathsf{T}}\\ &\quad +\sigma_s\mathbf{B}_s - \frac{\overline{d}_m\sigma_s^2 e^{\gamma\overline{d}_m}}{1-\eta_m}\cdot\mathbf{B}_s\mathbf{Q}_m^{-1}\mathbf{B}_s^{\mathsf{T}} - \frac{\sigma}{2}\cdot\mathbf{P}\end{aligned} \tag{20}$$

and $0 < \eta_i < 1$, $\omega_i = \gamma\lambda_{i2}/2 + n\upsilon_i/\gamma$, $\upsilon_i > 0$, $i = m, s$, $\upsilon = \min(\upsilon_m, \upsilon_s)$, and the function $\chi(\|\mathbf{u}\|)$ is

$$\chi(\|\mathbf{u}\|) = \frac{1}{2\upsilon}\cdot\|\mathbf{u}\|^2 + \sqrt{2n}\cdot\|\mathbf{u}\|.$$

Remark 6: In $\boldsymbol{\Psi}_{33}$, $\sigma = \sigma_m + \sigma_s$, where σ_m and σ_s are time dependent. Then, $\boldsymbol{\Psi}$ is positive semidefinite iff: 1) the gains in $\boldsymbol{\Psi}_{11}$ and $\boldsymbol{\Psi}_{22}$ are selected to make $\boldsymbol{\Psi}_{11}$ and $\boldsymbol{\Psi}_{22}$ positive semidefinite and 2) $\boldsymbol{\Psi}_{33}$ is positive semidefinite for any σ_m and σ_s. Because $\boldsymbol{\Psi}_{33}$ is quadratic in σ_m and σ_s, and zero when $\sigma_m = \sigma_s = 0$, there exist $\mathbf{B}_m$ and $\mathbf{B}_s$ matrices such that: 1) $\mathbf{B}_m - (1/2)\mathbf{P}$ and $\mathbf{B}_s - (1/2)\mathbf{P}$ are positive definite and 2) the second solution $(\hat{\sigma}_m, \hat{\sigma}_s)$ of the quadratic equation $\boldsymbol{\Psi}_{33}(\hat{\sigma}_m, \hat{\sigma}_s) = 0$ is positive. Then, selecting $\mathrm{sat}(\dot{q}_{ik})$ such that $0 < \gamma \le \min(\hat{\sigma}_m, \hat{\sigma}_s)$ makes $\boldsymbol{\Psi}_{33}$ positive semidefinite.

Remark 7: Equation (19) proposes for the first time the velocity-dependent gain σ in $\dot{V}$. The idea is inspired by the observation that σ quantifies the synchronization speed of the master–slave subsystem from its initial state under the perturbations of the user and environment forces together with their transmission. As will be shown in the proof of Theorem 2, if $\boldsymbol{\Psi}$ is made positive semidefinite by proper parameter selections, then: 1) σ implies the vanishing speed of $V(0)$ and 2) in the absence of external perturbations, σ also determines the synchronization speed of the system. Because synchronization is achieved by dissipating the kinetic energy and potential energy of the system, the synchronization speed should be related to the speed of energy consumption. By the controller design (18), the energy of the system is actually consumed by $-\mathbf{S}_i(\dot{\mathbf{q}}_i)\cdot(\mathbf{q}_i - \mathbf{q}_{jd})^{\mathsf{T}}\mathbf{B}_i(\mathbf{q}_i - \mathbf{q}_{jd}) - \mathbf{K}_i\dot{\mathbf{q}}_i$, where $i, j = m, s$ and $i \ne j$, which can be regarded as a type of nonlinear damping injection. Therefore, the energy consumption speed and, thus, the synchronization speed could be quantified by a parameter σ about robot velocities.

Proof: As in Section III-A, property P.2 can be used to show that

$$\begin{aligned}\dot{V}_k &\le \dot{\mathbf{q}}_m^{\mathsf{T}}(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) - \dot{\mathbf{q}}_m^{\mathsf{T}}\mathbf{K}_m\dot{\mathbf{q}}_m - \dot{\mathbf{q}}_m^{\mathsf{T}}\mathbf{P}\int_{t-d_s}^t \dot{\mathbf{q}}_s(\xi)d\xi\\ &\quad -\dot{\mathbf{q}}_m^{\mathsf{T}}\mathbf{P}(\mathbf{q}_m - \mathbf{q}_s) - \sigma_m\cdot(\mathbf{q}_m - \mathbf{q}_s)^{\mathsf{T}}\mathbf{B}_m(\mathbf{q}_m - \mathbf{q}_s)\\ &\quad -2\sigma_m\cdot(\mathbf{q}_m - \mathbf{q}_s)^{\mathsf{T}}\mathbf{B}_m\int_{t-d_s}^t \dot{\mathbf{q}}_s(\xi)d\xi\\ &\quad +\dot{\mathbf{q}}_s^{\mathsf{T}}(\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd}) - \dot{\mathbf{q}}_s^{\mathsf{T}}\mathbf{K}_s\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_s^{\mathsf{T}}\mathbf{P}\int_{t-d_m}^t \dot{\mathbf{q}}_m(\xi)d\xi\\ &\quad -\dot{\mathbf{q}}_s^{\mathsf{T}}\mathbf{P}(\mathbf{q}_s - \mathbf{q}_m) - \sigma_s\cdot(\mathbf{q}_s - \mathbf{q}_m)^{\mathsf{T}}\mathbf{B}_s(\mathbf{q}_s - \mathbf{q}_m)\\ &\quad -2\sigma_s\cdot(\mathbf{q}_s - \mathbf{q}_m)^{\mathsf{T}}\mathbf{B}_s\int_{t-d_m}^t \dot{\mathbf{q}}_m(\xi)d\xi.\end{aligned} \tag{21}$$

From Lemma L.1 in [34], it follows that:

$$\begin{aligned}&-\dot{\mathbf{q}}_m^{\mathsf{T}}\mathbf{P}\int_{t-d_s}^t \dot{\mathbf{q}}_s(\xi)d\xi - e^{-\gamma\overline{d}_s}\int_{t-d_s}^t \dot{\mathbf{q}}_s^{\mathsf{T}}(\xi)\mathbf{Q}_s\dot{\mathbf{q}}_s(\xi)d\xi\\ &\quad -2\sigma_m\cdot(\mathbf{q}_m - \mathbf{q}_s)^{\mathsf{T}}\mathbf{B}_m\int_{t-d_s}^t \dot{\mathbf{q}}_s(\xi)d\xi\end{aligned}$$

$$\leq \frac{\overline{d}_s \sigma_m^2 e^{\gamma \overline{d}_s}}{1-\eta_s} \cdot (\mathbf{q}_m - \mathbf{q}_s)^\top \mathbf{B}_m \mathbf{Q}_s^{-1} \mathbf{B}_m^\top (\mathbf{q}_m - \mathbf{q}_s) + \frac{\overline{d}_s e^{\gamma \overline{d}_s}}{4\eta_s} \cdot \dot{\mathbf{q}}_m^\top \mathbf{P} \mathbf{Q}_s^{-1} \mathbf{P}^\top \dot{\mathbf{q}}_m \quad (22)$$

and similarly that

$$-\dot{\mathbf{q}}_s^\top \mathbf{P} \int_{t-d_m}^{t} \dot{\mathbf{q}}_m(\xi) d\xi - e^{-\gamma \overline{d}_m} \int_{t-d_m}^{t} \dot{\mathbf{q}}_m^\top(\xi) \mathbf{Q}_m \dot{\mathbf{q}}_m(\xi) d\xi - 2\sigma_s \cdot (\mathbf{q}_s - \mathbf{q}_m)^\top \mathbf{B}_s \int_{t-d_m}^{t} \dot{\mathbf{q}}_m(\xi) d\xi$$
$$\leq \frac{\overline{d}_m \sigma_s^2 e^{\gamma \overline{d}_m}}{1-\eta_m} \cdot (\mathbf{q}_s - \mathbf{q}_m)^\top \mathbf{B}_s \mathbf{Q}_m^{-1} \mathbf{B}_s^\top (\mathbf{q}_s - \mathbf{q}_m) + \frac{\overline{d}_m e^{\gamma \overline{d}_m}}{4\eta_m} \cdot \dot{\mathbf{q}}_s^\top \mathbf{P} \mathbf{Q}_m^{-1} \mathbf{P}^\top \dot{\mathbf{q}}_s. \quad (23)$$

Then, combining the sum of (12), (13), and (21) with (11), (22), and (23) leads to

$$\begin{aligned}
\dot{V} \leq\ & \overline{d}_m \dot{\mathbf{q}}_m^\top \mathbf{Q}_m \dot{\mathbf{q}}_m + \frac{\overline{d}_s e^{\gamma \overline{d}_s}}{4\eta_s} \cdot \dot{\mathbf{q}}_m^\top \mathbf{P} \mathbf{Q}_s^{-1} \mathbf{P}^\top \dot{\mathbf{q}}_m - \dot{\mathbf{q}}_m^\top \mathbf{K}_m \dot{\mathbf{q}}_m \\
& + \frac{\gamma \lambda_{m2}}{2} \cdot \dot{\mathbf{q}}_m^\top \dot{\mathbf{q}}_m - \sigma_m \cdot (\mathbf{q}_m - \mathbf{q}_s)^\top \mathbf{B}_m (\mathbf{q}_m - \mathbf{q}_s) \\
& + \frac{\overline{d}_s \sigma_m^2 e^{\gamma \overline{d}_s}}{1-\eta_s} \cdot (\mathbf{q}_m - \mathbf{q}_s)^\top \mathbf{B}_m \mathbf{Q}_s^{-1} \mathbf{B}_m^\top (\mathbf{q}_m - \mathbf{q}_s) \\
& + \overline{d}_s \dot{\mathbf{q}}_s^\top \mathbf{Q}_s \dot{\mathbf{q}}_s + \frac{\overline{d}_m e^{\gamma \overline{d}_m}}{4\eta_m} \cdot \dot{\mathbf{q}}_s^\top \mathbf{P} \mathbf{Q}_m^{-1} \mathbf{P}^\top \dot{\mathbf{q}}_s - \dot{\mathbf{q}}_s^\top \mathbf{K}_s \dot{\mathbf{q}}_s \\
& + \frac{\gamma \lambda_{s2}}{2} \cdot \dot{\mathbf{q}}_s^\top \dot{\mathbf{q}}_s - \sigma_s \cdot (\mathbf{q}_s - \mathbf{q}_m)^\top \mathbf{B}_s (\mathbf{q}_s - \mathbf{q}_m) \\
& + \frac{\overline{d}_m \sigma_s^2 e^{\gamma \overline{d}_m}}{1-\eta_m} \cdot (\mathbf{q}_s - \mathbf{q}_m)^\top \mathbf{B}_s \mathbf{Q}_m^{-1} \mathbf{B}_s^\top (\mathbf{q}_s - \mathbf{q}_m) \\
& + \frac{\sigma}{2} \cdot (\mathbf{q}_m - \mathbf{q}_s)^\top \mathbf{P} (\mathbf{q}_m - \mathbf{q}_s) - \sigma \cdot \left(V_k + V_p\right) - \gamma \cdot V_d \\
& + \dot{\mathbf{q}}_m^\top (\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) + \dot{\mathbf{q}}_s^\top (\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd}).
\end{aligned} \quad (24)$$

Consider

$$\dot{\mathbf{q}}_m^\top (\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) = \sum_{k=1}^{n} \dot{q}_{mk} (\tau_{hk} + \tau_{edk}).$$

Case 1: If $|\dot{q}_{mk}| < \gamma/2n$, $\forall k = 1, \ldots, n$, it follows that:

$$\dot{\mathbf{q}}_m^\top (\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) \leq \sum_{k=1}^{n} |\dot{q}_{mk} (\tau_{hk} + \tau_{edk})| \leq \sigma_m \cdot \sum_{k=1}^{n} |\tau_{hk} + \tau_{ek}| \leq \sqrt{n} \cdot \sigma \cdot \|\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}\|.$$

Case 2: If $\exists k = 1, \ldots, n$ such that $|\dot{q}_{mk}| \geq \gamma/2n$, it follows that $\sigma \geq \sigma_m \geq \gamma/2n$, and that:

$$\dot{\mathbf{q}}_m^\top (\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) \leq \frac{\upsilon_m}{2\sigma} \cdot \dot{\mathbf{q}}_m^\top \dot{\mathbf{q}}_m + \frac{\sigma}{2\upsilon_m} \cdot \|\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}\|^2 \leq \frac{n\upsilon_m}{\gamma} \cdot \dot{\mathbf{q}}_m^\top \dot{\mathbf{q}}_m + \frac{\sigma}{2\upsilon_m} \cdot \|\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}\|^2.$$

Together, Cases 1 and 2 imply that

$$\dot{\mathbf{q}}_m^\top (\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) \leq \frac{n\upsilon_m}{\gamma} \cdot \dot{\mathbf{q}}_m^\top \dot{\mathbf{q}}_m + \sqrt{n} \cdot \sigma \cdot \|\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}\| + \frac{\sigma}{2\upsilon_m} \cdot \|\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}\|^2. \quad (25)$$

Similarly, it can be shown that

$$\dot{\mathbf{q}}_s^\top (\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd}) \leq \frac{n\upsilon_s}{\gamma} \cdot \dot{\mathbf{q}}_s^\top \dot{\mathbf{q}}_s + \sqrt{n} \cdot \sigma \cdot \|\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd}\| + \frac{\sigma}{2\upsilon_s} \cdot \|\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd}\|^2. \quad (26)$$

After substituting (25) and (26) in (24), the derivative of V can be bounded by (19). □

Theorem 2: The teleoperation system (1) in closed loop with the controller (18) is IOS with input $\mathbf{u}(t) = [(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed})^\top\ (\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd})^\top]^\top$, state $\mathbf{x}(t) = [\dot{\mathbf{q}}_m^\top\ \dot{\mathbf{q}}_s^\top\ (\mathbf{q}_m - \mathbf{q}_s)^\top]^\top$, and output $\mathbf{y}(t) = h(t, \mathbf{x}(t)) = \mathbf{x}(t)$ if the control gains $\mathbf{K}_i$, $\mathbf{B}_i$, and $\mathbf{P}$, the parameters $\mathbf{Q}_i$, η_i, and υ_i, and the nonlinear functions $\mathbf{S}_i(\dot{\mathbf{q}}_i)$, $i = m, s$ are selected such that the block diagonal matrix $\boldsymbol{\Psi}$ (19) is positive semidefinite.

Proof: For $\boldsymbol{\Psi}$ positive semidefinite, i.e., $-\mathbf{x}^\top \boldsymbol{\Psi} \mathbf{x} \leq 0$, $\forall \mathbf{x}$, (19) can be rewritten as

$$\dot{V} \leq -\sigma \cdot (V_k + V_p) - \gamma \cdot V_d + \sigma \cdot \chi(\|\mathbf{u}(t)\|).$$

From the definitions of σ and V_d, it follows that $\sigma \leq \gamma$ and that:

$$\dot{V} \leq -\sigma(t) \cdot V + \sigma(t) \cdot \chi(\|\mathbf{u}(t)\|). \quad (27)$$

By the comparison theorem, time-integration of $\dot{V}$ yields

$$\begin{aligned}
V &\leq e^{-\int_{t_0}^{t} \sigma(\xi) d\xi} \cdot V_0 + \int_{t_0}^{t} e^{-\int_{\xi}^{t} \sigma(\theta) d\theta} \cdot \sigma(\xi) \cdot \chi(\|\mathbf{u}(\xi)\|) d\xi \\
&\leq e^{-\int_{t_0}^{t} \sigma(\xi) d\xi} \cdot \sup_{t_0 \leq \tau \leq t} \chi(\|\mathbf{u}(\tau)\|) \cdot \int_{t_0}^{t} \sigma(\xi) \cdot e^{\int_{t_0}^{\xi} \sigma(\theta) d\theta} d\xi + e^{-\int_{t_0}^{t} \sigma(\xi) d\xi} \cdot V_0 \\
&= e^{-\int_{t_0}^{t} \sigma(\xi) d\xi} \cdot V_0 + \psi(t_0, t, \sigma) \cdot \sup_{t_0 \leq \tau \leq t} \chi(\|\mathbf{u}(\tau)\|)
\end{aligned} \quad (28)$$

where $V_0 = V(t_0)$, and

$$0 \leq \psi(t_0, \sigma) = 1 - e^{-\int_{t_0}^{t} \sigma(\xi) d\xi} \leq 1, \quad \forall t \geq t_0 \geq 0.$$

Similar to the proof Theorem 1, the definition of V in (7) leads to

$$\frac{\varrho_1}{2} \cdot \|\mathbf{x}\|^2 \leq V \leq \frac{\varrho_2}{2} \cdot \|\mathbf{x}\|_r^2 \quad (29)$$

where $\varrho_1 = \min(\lambda_{m1}, \lambda_{s1}, \underline{P})$ and $\varrho_2 = \max(\lambda_{m2}, \lambda_{s2}, \overline{P}) + \overline{d}_m^2 \overline{Q}_m + \overline{d}_s^2 \overline{Q}_s$. Together, (28) and (29) imply that

$$\|\mathbf{y}(t)\|^2 \leq \frac{1}{2} \cdot \left[\beta^2(\|\mathbf{x}_0\|_r, t - t_0) + \sup_{t_0 \leq \tau \leq t} \alpha^2 \cdot (\|\mathbf{u}(\tau)\|)\right] \quad (30)$$

and further that

$$\|\mathbf{y}(t)\| \leq \max\left\{\beta(\|\mathbf{x}_0\|_r, t - t_0), \sup_{t_0 \leq \tau \leq t} \alpha(\|\mathbf{u}(\tau)\|)\right\} \quad (31)$$

where

$$\alpha(\|\mathbf{u}(\tau)\|) = \sqrt{\frac{4}{\varrho_1}\cdot\chi(\|\mathbf{u}(\tau)\|)} \tag{32}$$

and

$$\beta(\|\mathbf{x}_0\|_r, t-t_0) = \sqrt{\frac{2\varrho_2}{\varrho_1}}\cdot\sqrt{e^{-\int_0^{t-t_0}\sigma(t_0+\xi)d\xi}}\cdot\|\mathbf{x}_0\|_r. \tag{33}$$

Because $\alpha(\cdot)$ in (32) and $\beta(\cdot, t-t_0)$ in (33) are of class $\mathcal{K}$ for each $t \geq t_0$, and $\beta(\|\mathbf{x}_0\|_r, \cdot)$ is decreasing with $\|\mathbf{x}_0\|_r$ fixed, (31) implies that the closed-loop system is RFC by definition D.1.

To prove the RGAOS property, let the input $\mathbf{u}(t) \equiv \mathbf{0}$. Then, condition 1) in definition D.3 is trivially satisfied from (31). Furthermore, given any $\epsilon > 0$ and $T \geq 0$, the selection $\delta = \delta(\epsilon) = ((\varrho_1/\varrho_2))^{(1/2)}\cdot\epsilon$ makes $\|\mathbf{y}(t)\| \leq \epsilon$ by (30) for all $t_0 \in [0, T]$, $\|\mathbf{x}_0\|_r \leq \delta$, and $t \geq t_0$. Therefore, it guarantees condition 2) of RGAOS. Condition 3) is guaranteed if $\lim_{t\to+\infty}\|\mathbf{y}(t)\| = 0$. Because (30) implies that

$$\lim_{t-t_0\to+\infty}\beta(\|\mathbf{x}_0\|_r, t-t_0) = 0$$
$$\Longrightarrow \lim_{t\to+\infty}\|\mathbf{y}(t)\| = 0$$

when $\mathbf{u}(t) \equiv \mathbf{0}$, it suffices to prove that $\beta(\|\mathbf{x}_0\|_r, t-t_0) \to 0$ as $t \to +\infty$. To this end, assume that $\beta(\|\mathbf{x}_0\|_r, t-t_0)$ does not tend to 0. Because $\beta(\|\mathbf{x}_0\|_r, \cdot)$ is continuous, decreasing and lower bounded by 0, there exists $0 < \hat{\beta} < \infty$ such that

$$\lim_{t-t_0\to+\infty}\beta(\|\mathbf{x}_0\|_r, t-t_0) = \hat{\beta}. \tag{34}$$

Because the derivative of $\beta(\|\mathbf{x}_0\|_r, t-t_0)$

$$\dot{\beta}(\|\mathbf{x}_0\|_r, t-t_0) = -\frac{1}{2}\cdot\beta(\|\mathbf{x}_0\|_r, t-t_0)\cdot\sigma(t)$$

is uniformly continuous, Barbalat's lemma implies that

$$\lim_{t-t_0\to+\infty}\dot{\beta}(\|\mathbf{x}_0\|_0, t-t_0) = 0$$

and $\sigma(t) \to 0$ by (34), and

$$\lim_{t\to+\infty}\dot{\mathbf{q}}_i(t) = \mathbf{0}, \quad i = m, s \tag{35}$$

by the definition of σ in Lemma 2. Because $\ddot{\mathbf{q}}_i$, $i = m, s$, are uniformly continuous, Barbalat's lemma also indicates that $\ddot{\mathbf{q}}_i(t) \to \mathbf{0}$ as $t \to +\infty$ and that

$$\lim_{t\to+\infty}\mathbf{q}_m(t) - \mathbf{q}_s(t) = \mathbf{0} \tag{36}$$

by the teleoperation dynamics (1). Then, (35) and (36) lead to $\lim_{t\to+\infty}\|\mathbf{y}(t)\| = 0$, which ensures condition 3). Thus, the system is nonuniformly in time RGAOS with $\mathbf{u}(t) \equiv \mathbf{0}$.

By theorem [54, T.2], the four-channel teleoperation system (1) in closed loop with the controller (18) is IOS with input $\mathbf{u}(t) = [(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed})^{\mathsf{T}}\ (\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd})^{\mathsf{T}}]^{\mathsf{T}}$, state $\mathbf{x}(t) = [\dot{\mathbf{q}}_m^{\mathsf{T}}\ \dot{\mathbf{q}}_s^{\mathsf{T}}\ (\mathbf{q}_m - \mathbf{q}_s)^{\mathsf{T}}]^{\mathsf{T}}$, and output $\mathbf{y}(t) = h(t, \mathbf{x}(t)) = \mathbf{x}(t)$. □

Remark 8: Although it employs the same Lyapunov–Krasovskii functional (7) as Section III-A, the proof of Theorem 2 requires [[54], T.2] rather than T.1 in [54] because σ defined in Lemma 2 has a twofold impact on the stability proof as follows.

1) Because σ depends on robot velocities $\dot{\mathbf{q}}_i$ and not on V, it is not trivial to prove that $\dot{V}$ in (27) satisfies condition (5).
2) Because σ may be integrable, i.e., $\int_{t_0}^{t}\sigma(\xi)d\xi < +\infty$, $\beta(\|\mathbf{x}_0\|_r, t-t_0)$ is not necessarily of class $\mathcal{KL}$ and (30) does not imply condition (3).

Therefore, theorem [54, T.1] and definition D.2 do not help prove Theorem 2. Nevertheless, V in (7) serves to show that the system is RFC and, together with Barbalat's lemma, to demonstrate that four-channel teleoperation is RGAOS with $\mathbf{u}(t) \equiv \mathbf{0}$.

C. Reduced-Order Hybrid Damping-Stiffness Adjustment

The nonsingular hybrid damping-stiffness adjustment in Section III-B smoothes the force feedback compared to the singular strategy in Section III-A. However, for sufficiently large position error, the hybrid control term can overwhelm the Proportional control term and thwart the master–slave coordination. This behavior, called sticking, can be avoided by moving the master and slave robots at the same joint space position before the teleoperation starts. For applications in which the initial master–slave coordination cannot be guaranteed, potential sticking should be avoided through control design. For stable and sticking-free four-channel teleoperation with time-varying delays, this section proposes the following reduced-order hybrid damping-stiffness adjustment strategy:

$$\begin{aligned}\boldsymbol{\tau}_m &= -[\mathbf{P} + \mathbf{S}(\dot{\mathbf{q}}_m, \mathbf{q}_m - \mathbf{q}_{sd})](\mathbf{q}_m - \mathbf{q}_{sd})\\ &\quad -\mathbf{K}_m\dot{\mathbf{q}}_m + \boldsymbol{\tau}_{ed} + \mathbf{g}_m\\ \boldsymbol{\tau}_s &= -[\mathbf{P} + \mathbf{S}(\dot{\mathbf{q}}_s, \mathbf{q}_s - \mathbf{q}_{md})](\mathbf{q}_s - \mathbf{q}_{md})\\ &\quad -\mathbf{K}_s\dot{\mathbf{q}}_s + \boldsymbol{\tau}_{hd} + \mathbf{g}_s\end{aligned} \tag{37}$$

where $\mathbf{S}(\dot{\mathbf{q}}_i, \mathbf{q}_i - \mathbf{q}_{jd}) = \varrho\cdot\text{sgn}(\dot{\mathbf{q}}_i)\cdot\text{sgn}(\mathbf{q}_i - \mathbf{q}_{jd})^{\mathsf{T}}$ with $i, j = \{m, s\}$ and $i \neq j$, $\text{sgn}(\mathbf{u}) = [\text{sgn}(u_1), \cdots, \text{sgn}(u_n)]^{\mathsf{T}}$ and $\text{sgn}(\cdot)$ the signum function; ϱ is a positive constant; and $\mathbf{P}$ and $\mathbf{K}_i$ are the positive diagonal gain matrices.

Remark 9: In (37), the nonlinear hybrid gains $\mathbf{S}(\dot{\mathbf{q}}_i, \mathbf{q}_i - \mathbf{q}_{jd})$ modify the conventional Proportional gain $\mathbf{P}$ to generate a position synchronization control term $-p_k(q_{ik} - q_{jdk}) - \varrho\cdot\text{sgn}(\dot{q}_{ik})\|\mathbf{q}_i - \mathbf{q}_{jd}\|_1$ for each joint k of each robot $i = m, s$. For sufficiently large position error along the joint space direction k, i.e., $p_k > n\varrho$, the conventional Proportional control term $-p_k(q_{ik} - q_{jdk})$ is larger than the reduced-order hybrid control term $-\varrho\cdot\text{sgn}(\dot{q}_{ik})\cdot\|\mathbf{q}_i - \mathbf{q}_{jd}\|$ and the controller can decrease the position error along this joint space direction. Hence, the reduced-order hybrid controller (37) can coordinate the master and slave robots even for initially large position error. This is an important advantage over the nonsingular controller in the previous section.

The closed-loop stability of the system (1) under the control of (37) is analyzed using the same Lyapunov–Krasovskii function as in (7)

Lemma 3: Let $\sigma = \min(\gamma, \sigma_m + \sigma_s)$, where

$$\sigma_i = \frac{\varphi}{V_p}\cdot\max_{k=1,\ldots,n}(|\dot{q}_{ik}|)\cdot\max_{k=1,\ldots,n}(|q_{mk} - q_{sk}|)$$

with $i = m, s$, $\gamma > 0$ and $\varphi > 0$. Define the same state $\mathbf{x}$ and input $\mathbf{u}$ as in Lemma 1. If the controller parameters are selected such that

$$\begin{aligned}\boldsymbol{\Psi}_m &= \mathbf{K}_m - \overline{d}_m \mathbf{Q}_m - \frac{\overline{d}_s e^{\gamma \overline{d}_s}}{4\eta_s}\mathbf{P}\mathbf{Q}_s^{-1}\mathbf{P}^\top \\ &\quad - \left[\frac{\gamma \lambda_{m2}}{2} + \frac{\upsilon_m}{2\gamma} + \frac{n\overline{d}_s \varrho^2 e^{\gamma \overline{d}_s}\cdot \mathrm{tr}(\mathbf{Q}_s^{-1})}{4(1-\eta_s)}\right]\cdot \mathbf{I} \succeq \mathbf{0} \\ \boldsymbol{\Psi}_s &= \mathbf{K}_s - \overline{d}_s \mathbf{Q}_s - \frac{\overline{d}_m e^{\gamma \overline{d}_m}}{4\eta_m}\mathbf{P}\mathbf{Q}_m^{-1}\mathbf{P}^\top \\ &\quad - \left[\frac{\gamma \lambda_{s2}}{2} + \frac{\upsilon_s}{2\gamma} + \frac{n\overline{d}_m \varrho^2 e^{\gamma \overline{d}_m}\cdot \mathrm{tr}(\mathbf{Q}_m^{-1})}{4(1-\eta_m)}\right]\cdot \mathbf{I} \succeq \mathbf{0} \\ \psi_i &= \varrho - \varphi - \frac{\upsilon_i \overline{P}}{4\varphi} \geq 0 \end{aligned} \tag{38}$$

where $\mathrm{tr}(\mathbf{Q}_i^{-1})$ is the trace of $\mathbf{Q}_i^{-1}$, $0 < \eta_i < 1$ and $\upsilon_i > 0$, the derivative of the Lyapunov–Krasovskii function (7) can then be upper bounded by

$$\dot{V} \leq -\sigma \cdot (V_k + V_p) - \gamma \cdot V_d + \sigma \cdot \chi(\|\mathbf{u}\|) \tag{39}$$

where $\upsilon = \min(\upsilon_m, \upsilon_s)$ and

$$\chi(\|\mathbf{u}\|) = \frac{1}{2\upsilon}\cdot \|\mathbf{u}\|^2.$$

Proof: By property P.2, the time derivatives of V_k is

$$\begin{aligned}\dot{V}_k &= -\dot{\mathbf{q}}_m^\top \mathbf{K}_m \dot{\mathbf{q}}_m + \dot{\mathbf{q}}_m^\top(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) - \dot{\mathbf{q}}_m^\top \mathbf{P}(\mathbf{q}_m - \mathbf{q}_s) \\ &\quad -\dot{\mathbf{q}}_s^\top \mathbf{K}_s \dot{\mathbf{q}}_s + \dot{\mathbf{q}}_s^\top(\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd}) - \dot{\mathbf{q}}_s^\top \mathbf{P}(\mathbf{q}_s - \mathbf{q}_m) \\ &\quad -\dot{\mathbf{q}}_m^\top \mathbf{P}\int_{t-d_s}^t \dot{\mathbf{q}}_s(\xi)d\xi - \dot{\mathbf{q}}_s^\top \mathbf{P}\int_{t-d_m}^t \dot{\mathbf{q}}_m(\xi)d\xi \\ &\quad -\dot{\mathbf{q}}_m^\top \mathbf{S}(\dot{\mathbf{q}}_m, \mathbf{q}_m - \mathbf{q}_{sd})\cdot(\mathbf{q}_m - \mathbf{q}_{sd}) \\ &\quad -\dot{\mathbf{q}}_s^\top \mathbf{S}(\dot{\mathbf{q}}_s, \mathbf{q}_s - \mathbf{q}_{md})\cdot(\mathbf{q}_s - \mathbf{q}_{md}). \end{aligned} \tag{40}$$

The definition of the reduced-order hybrid gains $\mathbf{S}(\dot{\mathbf{q}}_i, \mathbf{q}_i - \mathbf{q}_{jd})$ leads to the following bounding of the hybrid control terms of the master and slave robots

$$\begin{aligned}&-\dot{\mathbf{q}}_m^\top \mathbf{S}(\dot{\mathbf{q}}_m, \mathbf{q}_m - \mathbf{q}_{sd})\cdot(\mathbf{q}_m - \mathbf{q}_{sd}) \\ &\quad = -\varrho\cdot \dot{\mathbf{q}}_m^\top \cdot \mathrm{sgn}(\dot{\mathbf{q}}_m)\cdot \mathrm{sgn}(\mathbf{q}_m - \mathbf{q}_{sd})^\top \cdot (\mathbf{q}_m - \mathbf{q}_{sd}) \\ &\quad = -\varrho \cdot \left(\sum_{k=1}^n |\dot{q}_{mk}|\right)\cdot\left(\sum_{k=1}^n |q_{mk} - q_{sdk}|\right) \\ &\quad \leq -\varrho \cdot \left(\sum_{k=1}^n |\dot{q}_{mk}|\right)\cdot\left(\sum_{k=1}^n |q_{mk} - q_{sk}|\right) \\ &\qquad + \varrho\cdot\sum_{k=1}^n \left(|\dot{\mathbf{q}}_{mk}|^\top \int_{t-d_s}^t |\dot{\mathbf{q}}_s(\xi)|d\xi\right) \end{aligned} \tag{41}$$

and similarly

$$\begin{aligned}&-\dot{\mathbf{q}}_s^\top \mathbf{S}(\dot{\mathbf{q}}_s, \mathbf{q}_s - \mathbf{q}_{md})\cdot(\mathbf{q}_s - \mathbf{q}_{md}) \\ &\quad \leq -\varrho \cdot \left(\sum_{k=1}^n |\dot{q}_{sk}|\right)\cdot\left(\sum_{k=1}^n |q_{sk} - q_{mk}|\right) \\ &\qquad + \varrho\cdot\sum_{k=1}^n \left(|\dot{\mathbf{q}}_{sk}|^\top \int_{t-d_m}^t |\dot{\mathbf{q}}_m(\xi)|d\xi\right) \end{aligned} \tag{42}$$

where $\dot{\mathbf{q}}_{ik} = [\dot{q}_{ik}, \dot{q}_{ik}, \ldots, \dot{q}_{ik}]^\top$, $i = m, s$.

Furthermore, Lemma L.1 leads to

$$\begin{aligned}&\varrho\cdot\sum_{k=1}^n |\dot{\mathbf{q}}_{mk}|^\top \int_{t-d_s}^t |\dot{\mathbf{q}}_s(\xi)|d\xi - \dot{\mathbf{q}}_m^\top \mathbf{P}\int_{t-d_s}^t \dot{\mathbf{q}}_s(\xi)d\xi \\ &\quad - e^{-\gamma \overline{d}_s}\int_{t-d_s}^t \dot{\mathbf{q}}_s^\top(\xi)\mathbf{Q}_s\dot{\mathbf{q}}_s(\xi)d\xi \\ &= -\dot{\mathbf{q}}_m^\top \mathbf{P}\int_{t-d_s}^t \dot{\mathbf{q}}_s(\xi)d\xi - \eta_s e^{-\gamma\overline{d}_s}\int_{t-d_s}^t \dot{\mathbf{q}}_s^\top(\xi)\mathbf{Q}_s\dot{\mathbf{q}}_s(\xi)d\xi \\ &\quad + \varrho\cdot\sum_{k=1}^n |\dot{\mathbf{q}}_{mk}|^\top \int_{t-d_s}^t |\dot{\mathbf{q}}_s(\xi)|d\xi \\ &\quad - (1-\eta_s)\cdot e^{-\gamma\overline{d}_s}\int_{t-d_s}^t \dot{\mathbf{q}}_s^\top(\xi)\mathbf{Q}_s\dot{\mathbf{q}}_s(\xi)d\xi \\ &\leq \frac{\overline{d}_s\varrho^2 e^{\gamma\overline{d}_s}}{4(1-\eta_s)}\cdot\left(\sum_{i=1}^n\sum_{j=1}^n |\dot{\mathbf{q}}_{mi}|^\top \mathbf{Q}_s^{-1}|\dot{\mathbf{q}}_{mj}|\right) \\ &\quad + \frac{\overline{d}_s e^{\gamma\overline{d}_s}}{4\eta_s}\cdot \dot{\mathbf{q}}_m^\top \mathbf{P}\mathbf{Q}_s^{-1}\mathbf{P}^\top \dot{\mathbf{q}}_m \\ &\leq \frac{n\overline{d}_s\varrho^2 e^{\gamma\overline{d}_s}\cdot\mathrm{tr}(\mathbf{Q}_s^{-1})}{4(1-\eta_s)}\cdot \dot{\mathbf{q}}_m^\top\dot{\mathbf{q}}_m + \frac{\overline{d}_s e^{\gamma\overline{d}_s}}{4\eta_s}\cdot \dot{\mathbf{q}}_m^\top \mathbf{P}\mathbf{Q}_s^{-1}\mathbf{P}^\top\dot{\mathbf{q}}_m \end{aligned} \tag{43}$$

for the master robot and, similarly, to

$$\begin{aligned}&\varrho\cdot\sum_{k=1}^n |\dot{\mathbf{q}}_{sk}|^\top \int_{t-d_m}^t |\dot{\mathbf{q}}_m(\xi)|d\xi - \dot{\mathbf{q}}_s^\top \mathbf{P}\int_{t-d_m}^t \dot{\mathbf{q}}_m(\xi)d\xi \\ &\quad - e^{-\gamma \overline{d}_m}\int_{t-d_m}^t \dot{\mathbf{q}}_m^\top(\xi)\mathbf{Q}_m\dot{\mathbf{q}}_m(\xi)d\xi \\ &\leq \frac{n\overline{d}_m\varrho^2 e^{\gamma\overline{d}_m}\cdot\mathrm{tr}(\mathbf{Q}_m^{-1})}{4(1-\eta_m)}\cdot \dot{\mathbf{q}}_s^\top\dot{\mathbf{q}}_s + \frac{\overline{d}_m e^{\gamma\overline{d}_m}}{4\eta_m}\cdot \dot{\mathbf{q}}_s^\top \mathbf{P}\mathbf{Q}_m^{-1}\mathbf{P}^\top\dot{\mathbf{q}}_s \end{aligned} \tag{44}$$

for the slave robot, and to

$$\begin{aligned}\sigma\cdot(V_k + V_p) &\leq \sum_{i=m,s}\frac{\gamma\lambda_{i2}}{2}\cdot\dot{\mathbf{q}}_i^\top\dot{\mathbf{q}}_i \\ &\quad + \sum_{i=m,s}\varphi\cdot\max_{k=1,\ldots,n}(|\dot{q}_{ik}|) \\ &\quad \cdot \max_{k=1,\ldots,n}(|q_{mk} - q_{sk}|). \end{aligned} \tag{45}$$

The parameter σ is state dependent and switches between γ and $\sigma_m + \sigma_s$ over time. Its two possible behaviors and the constraints they impose on the design of the reduced-order hybrid damping-stiffness adjustment strategy (37) are analyzed next.

Case 1: Consider the teleoperation system at a time instant when $\sigma = \sigma_m + \sigma_s$. Then, the power due to the external operator and environment forces can be bounded by

$$\begin{aligned}&\dot{\mathbf{q}}_m^\top(\boldsymbol{\tau}_h + \boldsymbol{\tau}_{ed}) + \dot{\mathbf{q}}_s^\top(\boldsymbol{\tau}_e + \boldsymbol{\tau}_{hd}) \\ &\leq \sum_{i=m,s}\frac{\upsilon_i}{2\sigma}\cdot\dot{\mathbf{q}}_i^\top\dot{\mathbf{q}}_i + \frac{\sigma}{2\upsilon}\cdot\|\mathbf{u}\|^2 \\ &\leq \sum_{i=m,s}\frac{\upsilon_i\overline{P}}{4\varphi}\frac{\left(\sum_{k=1}^n \dot{q}_{ik}^2\right)\left(\sum_{k=1}^n |q_{mk}-q_{sk}|^2\right)}{\max_{k=1,\ldots,n}(|\dot{q}_{ik}|)\cdot\max_{k=1,\ldots,n}(|q_{mk}-q_{sk}|)} \end{aligned}$$

$$\begin{aligned}&+\frac{\sigma}{2\upsilon}\cdot\|\mathbf{u}\|^2\\ &\le \sum_{i=m,s}\frac{\upsilon_i\overline{P}}{4\varphi}\left(\sum_{k=1}^{n}|\dot{q}_{ik}|\right)\left(\sum_{k=1}^{n}|q_{mk}-q_{sk}|\right)+\frac{\sigma}{2\upsilon}\cdot\|\mathbf{u}\|^2\\ &= \sum_{i=m,s}\frac{\upsilon_i\overline{P}}{4\varphi}\sum_{k=1}^{n}\sum_{l=1}^{n}|\dot{q}_{ik}|\cdot|q_{ml}-q_{sl}|+\sigma\cdot\chi(\|\mathbf{u}\|). \qquad (46)\end{aligned}$$

A bound on the time derivative of V can be derived using the sum of (12), (13), and (40) together with (41)–(46)

$$\begin{aligned}\dot{V} &\le -\sigma\cdot(V_k+V_p)-\gamma\cdot V_d+\sigma\cdot\chi(\|\mathbf{u}\|)\\ &\quad-\sum_{i=m,s}\left(\dot{\mathbf{q}}_i^{\mathsf{T}}\boldsymbol{\Psi}_i'\dot{\mathbf{q}}_i+\psi_i\sum_{k=1}^{n}\sum_{l=1}^{n}|\dot{q}_{ik}|\cdot|q_{ml}-q_{sl}|\right)\end{aligned}$$

where

$$\begin{aligned}\boldsymbol{\Psi}_m' &= \mathbf{K}_m-\overline{d}_m\mathbf{Q}_m-\frac{\overline{d}_s e^{\gamma\overline{d}_s}}{4\eta_s}\mathbf{P}\mathbf{Q}_s^{-1}\mathbf{P}^{\mathsf{T}}\\ &\quad-\left[\frac{\gamma\lambda_{m2}}{2}+\frac{n\overline{d}_s\varrho^2 e^{\gamma\overline{d}_s}\cdot\mathrm{tr}(\mathbf{Q}_s^{-1})}{4(1-\eta_s)}\right]\cdot\mathbf{I}\\ \boldsymbol{\Psi}_s' &= \mathbf{K}_s-\overline{d}_s\mathbf{Q}_s-\frac{\overline{d}_m e^{\gamma\overline{d}_m}}{4\eta_m}\mathbf{P}\mathbf{Q}_m^{-1}\mathbf{P}^{\mathsf{T}}\\ &\quad-\left[\frac{\gamma\lambda_{s2}}{2}+\frac{n\overline{d}_m\varrho^2 e^{\gamma\overline{d}_m}\cdot\mathrm{tr}(\mathbf{Q}_m^{-1})}{4(1-\eta_m)}\right]\cdot\mathbf{I}.\end{aligned}$$

Choosing parameters such that $\boldsymbol{\Psi}_i'\succeq\mathbf{0}$ and $\psi_i\ge 0$ for $i=m,s$ leads to (39).

Case 2: Consider the teleoperation system at a time instant when $\sigma=\gamma$. Then, the power due to the external operator and environment forces at the master and slave sides can be bounded by

$$\begin{aligned}&\dot{\mathbf{q}}_m^{\mathsf{T}}(\boldsymbol{\tau}_h+\boldsymbol{\tau}_{ed})+\dot{\mathbf{q}}_s^{\mathsf{T}}(\boldsymbol{\tau}_e+\boldsymbol{\tau}_{hd})\\ &\qquad\le\sum_{i=m,s}\frac{\upsilon_i}{2\gamma}\cdot\dot{\mathbf{q}}_m^{\mathsf{T}}\dot{\mathbf{q}}_m+\sigma\cdot\chi(\|\mathbf{u}\|). \qquad (47)\end{aligned}$$

A bound on the time derivative of V can be derived using the sum of (12), (13), and (40) together with (41)–(45) and (47)

$$\begin{aligned}\dot{V} &\le -\sigma\cdot(V_k+V_p)-\gamma\cdot V_d+\sigma\cdot\chi(\|\mathbf{u}\|)\\ &\quad-\sum_{i=m,s}\left(\dot{\mathbf{q}}_i^{\mathsf{T}}\boldsymbol{\Psi}_i\dot{\mathbf{q}}_i+\psi_i'\sum_{k=1}^{n}\sum_{l=1}^{n}|\dot{q}_{ik}|\cdot|q_{ml}-q_{sl}|\right)\end{aligned}$$

where $\psi_i'=\varrho-\varphi$. Choosing parameters such that $\boldsymbol{\Psi}_i\succeq\mathbf{0}$ and $\psi_i'\ge 0$ for $i=m,s$ leads to (39).

The above-mentioned analysis of the closed-loop teleoperation system indicates that, although σ is state dependent and switches between γ and $\sigma_m+\sigma_s$ over time, if the controller parameters are selected such that $\boldsymbol{\Psi}_i=\min\{\boldsymbol{\Psi}_i,\boldsymbol{\Psi}_i'\}\succeq\mathbf{0}$ and $\psi_i=\min\{\psi_i,\psi_i'\}\ge 0$, $i=m,s$, then the Lyapunov–Krasovskii function (7) satisfies (39). □

Theorem 3: The teleoperation system (1) in closed loop with the controller (37) is IOS with input $\mathbf{u}(t)=[(\boldsymbol{\tau}_h+\boldsymbol{\tau}_{ed})^{\mathsf{T}}\ (\boldsymbol{\tau}_e+\boldsymbol{\tau}_{hd})^{\mathsf{T}}]^{\mathsf{T}}$, state $\mathbf{x}(t)=[\dot{\mathbf{q}}_m^{\mathsf{T}}\ \dot{\mathbf{q}}_s^{\mathsf{T}}\ (\mathbf{q}_m-\mathbf{q}_s)^{\mathsf{T}}]^{\mathsf{T}}$, and output $\mathbf{y}(t)=h(t,\mathbf{x}(t))=\mathbf{x}(t)$ if the control gains $\mathbf{K}_i$, $\mathbf{P}$, and ϱ and the parameters $\mathbf{Q}_i$, η_i, υ_i, γ, and ϵ are selected such that $\boldsymbol{\Psi}_i\succeq\mathbf{0}$ and $\psi_i\ge 0$ for $i=m,s$.

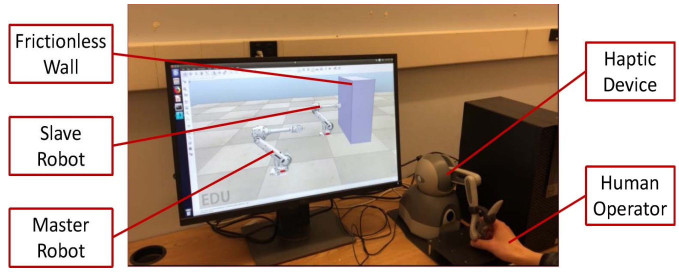

Fig. 1. Hardware-in-the-loop experimental setup.

The proof is similar to the proof of Theorem 2, and is omitted here to save space.

Remark 10: The design procedure for the reduced-order hybrid four-channel controller in (37) is performed as follows: after choosing the Proportional gain $\mathbf{P}$, the parameters ϱ, φ and υ_i are selected small enough to satisfy $\psi_i\ge 0$ and $\varrho<np_k$ for $i=m,s$ and $k=1,\ldots,n$; then the parameters $\mathbf{Q}_i$, γ, and η_i are chosen such that the damping $\mathbf{K}_i$ can be selected sufficiently small to guarantee that $\boldsymbol{\Psi}_i$ is positive semidefinite. This design procedure is simpler than the procedure in Section III-B, and is another advantage of the reduced-order strategy over the nonsingular one.

D. Discussion of IOS Teleoperation

In delay-free teleoperation, conventional P + d coordination control behaves as it does in single robot control: the P term forms a virtual spring between the master and slave, and the local d terms form dampers between each robot and the ground. In the absence of perturbations, the virtual spring drives the robots to each other. Its potential energy is converted to kinetic energy of the two robots, which the local dampers dissipate. The energy of the closed-loop teleoperation system with P + d control asymptotically tends to zero and the master and slave converge to the same position. In the presence of perturbing operator and environment forces, the motion of the two robots is driven by the virtual spring, the local dampers, and the external perturbations themselves. If the perturbing forces inject limited energy, or the local dampers maintain the total energy of the closed-loop teleoperation system bounded, the P + d coordination guarantees bounded velocities of, and position error between, the master and slave robots, with steady-state error dependent on the P gain.

In teleoperation with time-varying delays, conventional P+d coordination still acts as local dampers at the master and slave, but the Proportional control term becomes a distorted spring. The delays distort the master and slave Proportional terms $\mathbf{P}(\mathbf{q}_m-\mathbf{q}_{sd})$ and $\mathbf{P}(\mathbf{q}_s-\mathbf{q}_{md})$, and threaten stability by injecting energy in the closed-loop system. However, because the distortions depend only on the velocity of the remote robot, the local damping can be designed to serve double purpose: 1) to bound the velocity of the local robot and, thus, the energy of the distortions that bounded time-varying delays inject at the remote robot during a time interval equal to the maximum delay and 2) to dissipate the energy injected by the delay-induced distortions at the local robot. Then, the effective

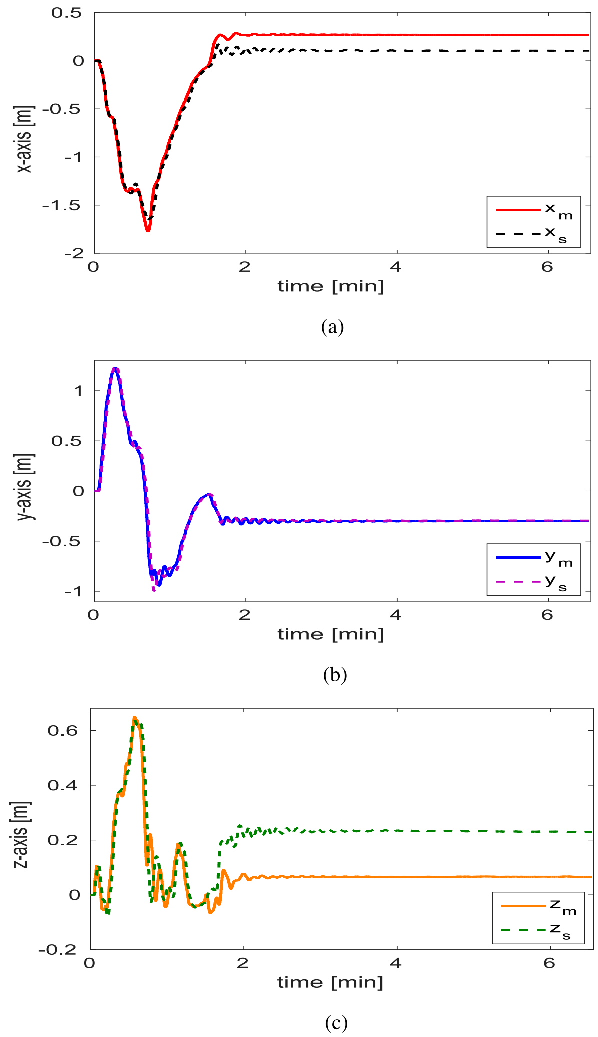

Fig. 2. Task space positions of the master and slave end-effectors under P + d control with no force exchange. (a) x-axis positions: master (x_m) and slave (x_s). (b) y-axis positions: master (y_m) and slave (y_s). (c) z-axis positions: master (z_m) and slave (z_s).

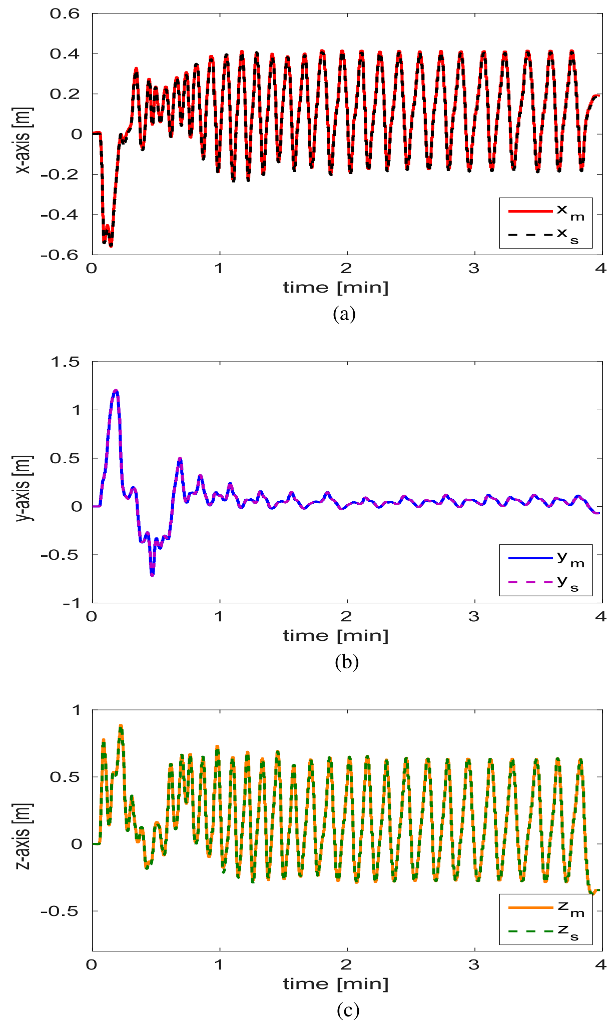

Fig. 3. Task space positions of the master and slave end-effectors under P + d control with delayed force transmission. (a) x-axis positions: master (x_m) and slave (x_s). (b) y-axis positions: master (y_m) and slave (y_s). (c) z-axis positions: master (z_m) and slave (z_s).

Proportional control can act as a virtual spring connecting the master and slave.

If the external operator and environment forces are not passive, they act as perturbations for the master and slave robots. In this paper, the torques due to the external forces $\boldsymbol{\tau}_h$ and $\boldsymbol{\tau}_e$ and their transmission to the other side $\boldsymbol{\tau}_{hd}$ and $\boldsymbol{\tau}_{ed}$ are called perturbations to distinguish their effects from those of the distortions $\mathbf{P}(\mathbf{q}_m - \mathbf{q}_{md})$ and $\mathbf{P}(\mathbf{q}_s - \mathbf{q}_{sd})$ induced in the Proportional control by the time-varying delays. Nevertheless, these perturbations should not be rejected because the master and slave robots need to be partially controlled by the operator and the environment. To achieve robust position tracking between the master and the slave without rejecting the operator and environment perturbations, this paper has designed controllers that render four-channel teleoperation IOS. In particular, the three proposed controllers use hybrid damping-stiffness adjustments in their position coordination channels to robustify the system: 1) to distortions in coordination forces and 2) to perturbations in the transmitted operator and environment forces, both caused by time-varying delays in the communications. By definition D.2, IOS four-channel teleoperation guarantees that the master and slave velocities do not escape in finite time, and that the two robots are coupled through Proportional control. It also guarantees the classical result in robot force control [56]: when both robots are at rest and the slave is in contact with the environment, the torques due to the operator and environment forces are constant and balance each other, and the position error between the master and the slave vanishes.

IV. Experimental Validation

This section contrasts four-channel teleoperation with the three hybrid damping-stiffness adjustment strategies to conventional two-channel P+d coordination through hardware-in-the-loop experiments. The experimental setup includes a Geomagic Touch haptic device, MATLAB/Simulink, and the robotic simulator V-REP by Robot Operating System (see Fig. 1). In V-REP, the simulated master and slave are ABB IRB 4600 industrial robots, and the simulated wall is frictionless. Task space Proportional plus damping control connects the master to the haptic device and enables the

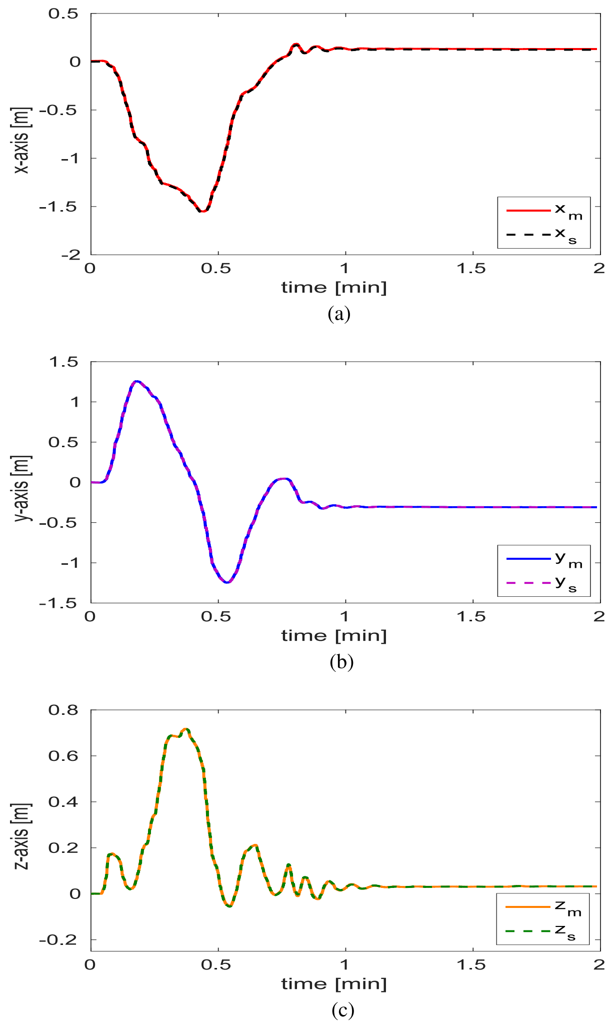

Fig. 4. Task space positions of the master and slave end-effectors under four-channel hybrid damping-stiffness adjustment control (6). (a) X-axis positions: master (x_m) and slave (x_s). (b) Y-axis positions: master (y_m) and slave (y_s). (c) Z-axis positions: master (z_m) and slave (z_s).

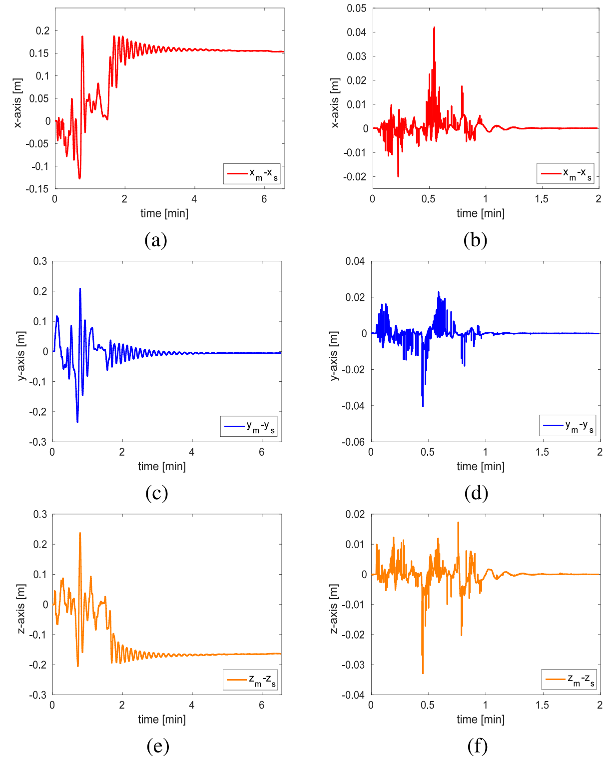

Fig. 5. Position tracking errors of the master and slave end-effectors in (a), (c), and (e) two-channel teleoperation with P+d control and in (b), (d), and (f) four-channel teleoperation with hybrid damping-stiffness adjustment. (a) $x_m - x_s$ shown in Fig. 2(a). (b) $x_m - x_s$ shown in Fig. 4(a). (c) $y_m - y_s$ shown in Fig.2(b). (d) $y_m - y_s$ shown in Fig. 4(b). (e) $z_m - z_s$ shown in Fig. 2(c). (f) $z_m - z_s$ shown in Fig. 4(c).

human operator to apply a force on the end-effector of the master by moving the haptic device. The joint space master and slave controllers are implemented in MATLAB/Simulink. Time-varying delays $d_i \leq 5$ ms, $i = m, s$ affect the communications between the two simulated robots. For ease of corresponding the plots to the video of the experimental teleoperations at https://youtu.be/c5yWdHLt1lM, this section presents the task space position tracking performance of the simulated four-channel teleoperator under the control of different strategies.

Case 1 (Instability Induced by Delayed Force Transmission): Teleoperation under classical P+d control [53] and without force exchange, that is,

$$\begin{aligned}\boldsymbol{\tau}_m &= -\mathbf{P}(\mathbf{q}_m - \mathbf{q}_{sd}) - \mathbf{K}_m\dot{\mathbf{q}}_m + \mathbf{g}_m\\ \boldsymbol{\tau}_s &= -\mathbf{P}(\mathbf{q}_s - \mathbf{q}_{md}) - \mathbf{K}_s\dot{\mathbf{q}}_s + \mathbf{g}_s\end{aligned}$$

can be stabilized by selecting $\mathbf{P} = 2 \times 10^5$ $\mathbf{I}$ and $\mathbf{K}_m = \mathbf{K}_s = 1000$ $\mathbf{I}$. Fig. 2 illustrates the robust master–slave synchronization during free motion, and large position error during slave-wall contact (time = 2–6.5 min).

However, the teleoperation becomes unstable when delayed force exchanges are employed in control, that is,

$$\begin{aligned}\boldsymbol{\tau}_m &= -\mathbf{P}(\mathbf{q}_m - \mathbf{q}_{sd}) - \mathbf{K}_m\dot{\mathbf{q}}_m + \boldsymbol{\tau}_{ed} + \mathbf{g}_m\\ \boldsymbol{\tau}_s &= -\mathbf{P}(\mathbf{q}_s - \mathbf{q}_{md}) - \mathbf{K}_s\dot{\mathbf{q}}_s + \boldsymbol{\tau}_{hd} + \mathbf{g}_s\end{aligned}$$

where $\mathbf{P}$ and $\mathbf{K}_m = \mathbf{K}_s$ are kept unchanged (see Fig. 3) when the slave touches the wall, the master and slave enter a limit cycle and the slave cannot maintain the contact with the wall (time = 1–4 min). In contrast, Cases 2–4 will validate the robust stability of four-channel teleoperation under the hybrid strategies proposed in this paper.

Case 2 (Hybrid Damping-Stiffness Adjustment): After choosing $\gamma = 0.01$, $\eta_i = 0.5$, $\upsilon_i = 1$, and $\mathbf{Q}_i = 10^5$ $\mathbf{I}$, the selection $\mathbf{P} = 2\times10^5$ $\mathbf{I}$, $\mathbf{K}_i = 1500$ $\mathbf{I}$, and $\mathbf{B}_i = 1000$ $\mathbf{I}$, $i = m, s$, makes $\boldsymbol{\Psi}$ in (9) positive semidefinite. Fig. 4 depicts the position tracking performance of four-channel teleoperation with this hybrid damping-stiffness adjustment. It shows tight master–slave synchronization, both during free motion (time = 0–1 min) and during slave contact with the wall (time = 1–2 min).

Fig. 5shows the comparision of the master and slave position tracking errors in two-channel teleoperation with P + d control (Fig. 2), and in four-channel teleoperation with hybrid damping-stiffness adjustment (Fig. 4). Along all three axes, hybrid four-channel control reduces the maximum errors by an order of magnitude compared to two-channel

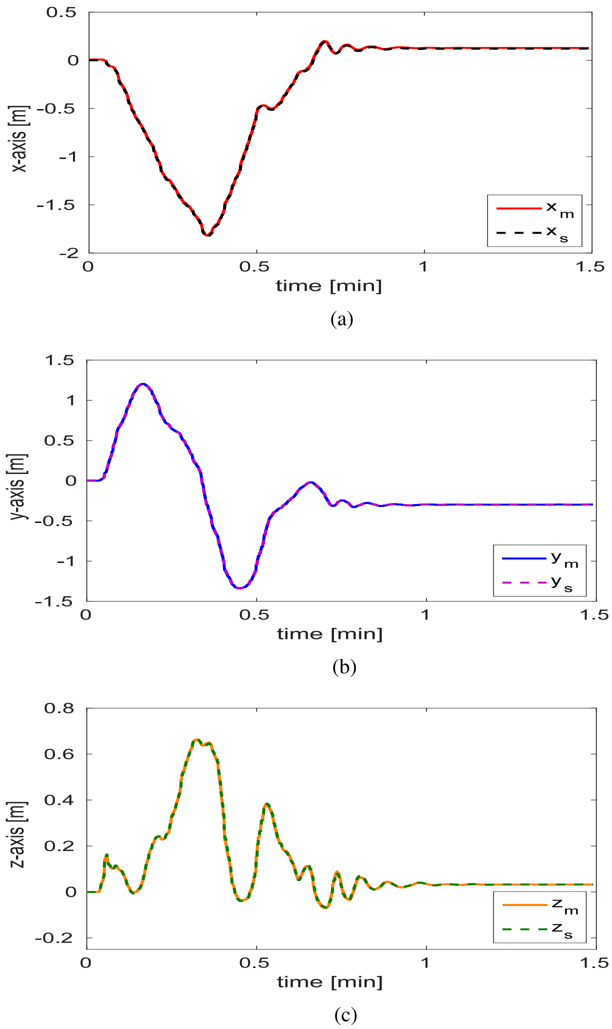

Fig. 6. Task space positions of the master and slave end-effectors during four-channel teleoperation with nonsingular hybrid adjustment (18). (a) x-axis positions: master (x_m) and slave (x_s). (b) y-axis positions: master (y_m) and slave (y_s). (c) z-axis positions: master (z_m) and slave (z_s).

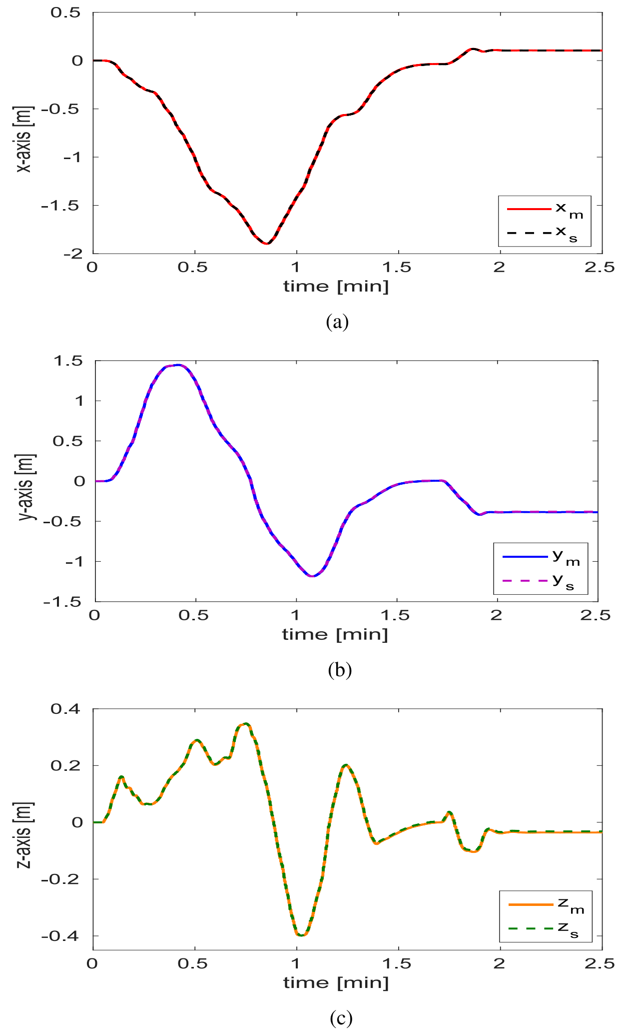

Fig. 7. Task space positions of master and slave end-effectors during four-channel teleoperation with reduced-order hybrid adjustment (37). (a) x-axis positions: master (x_m) and slave (x_s). (b) y-axis positions: master (y_m) and slave (y_s). (c) z-axis positions: master (z_m) and slave (z_s).

P + d control. During slave-wall contact, position tracking errors exist as shown in Fig. 2 and are eliminated as shown in Fig. 4.

Case 3 (Nonsingular Hybrid Adjustment): Fig. 6 plots the four-channel teleoperation with nonsingular hybrid adjustment (18). After choosing $\gamma = 0.1$, $\upsilon_i = 0.001$, $\eta_i = 0.8$, $\mathbf{Q}_i = 10^5\ \mathbf{I}$, the selection $\mathbf{P} = 2 \times 10^5\ \mathbf{I}$, $\mathbf{K}_i = 1500\ \mathbf{I}$, and $\mathbf{B}_i = \mathbf{P}$ makes $\boldsymbol{\Psi}$ in (20) positive semidefinite. Like the hybrid adjustment strategy (6), the nonsingular hybrid controller can also tightly constrain the master and slave robots, both during free motion and during slave-wall contact.

Case 4 (Reduced-Order Hybrid Adjustment): Fig. 7 shows the four-channel teleoperation with the reduced-order hybrid strategy (37). Let $\mathbf{P} = 2 \times 10^5\ \mathbf{I}$, $\varphi = 0.01$, $\varrho = 100$, $\upsilon_i = 10^{-7}$, $\eta_i = 0.8$, and $\mathbf{Q}_i = 10^6\ \mathbf{I}$, $i = m, s$. The selection $\gamma = 1$ makes ψ_i nonnegative, and $\mathbf{K}_i = 1500\ \mathbf{I}$ renders $\boldsymbol{\Psi}_i$ positive semidefinite. Compared to the nonsingular hybrid strategy, Fig. 7 indicates that the reduced-order hybrid control practically attenuates vibrations upon contact.

V. Conclusion

To guarantee robustly stable four-channel teleoperation with time-varying delays, this paper has proposed a hybrid damping-stiffness adjustment strategy for the master–slave position coordination channels. The hybrid strategy augments the conventional four-channel controller with a nonlinear term that combines local velocity and the master–slave position error at each robot side. Four-channel teleoperation with the hybrid adjustment couples the master to the slave and, implicitly, the human operator to the environment, tightly. However, the hybrid control terms are singular at zero velocities and may cause spikes in the control torques and chatter in the robot velocities. In practical implementation, they may reduce performance and even destroy the actuators and the robots. To overcome this limitation, this paper has introduced a second, nonsingular hybrid damping-stiffness adjustment strategy. The nonsingular controller employs saturation to eliminate the singularities and, with them, the torque spikes at zero velocities. Being quadratic in the position error, the nonsingular hybrid control terms may overwhelm the Proportional control terms and thwart the master–slave coordination. To guarantee the coordination of the two robots, however, large their position error may become, this paper has presented a third, reduced-order hybrid damping-stiffness adjustment strategy, whose hybrid control term is linear in position error. Future work will investigate how to distinguish the desirable from the harmful components of the delayed perturbing operator and environment forces.

REFERENCES

[1] N. Chopra, M. W. Spong, R. Ortega, and N. E. Barabanov, "Position and force tracking in bilateral teleoperation," in *Advances in Communication Control Networks*. Berlin, Germany: Springer, 2005, pp. 269–280.

[2] P. F. Hokayem and M. W. Spong, "Bilateral teleoperation: An historical survey," *Automatica*, vol. 42, no. 12, pp. 2035–2057, 2006.

[3] G. A. Christiansson, "An experimental study of haptic feedback in a teleoperated assembly task," *J. Comput. Inf. Sci. Eng.*, vol. 8, no. 4, 2008, Art. no. 041003.

[4] J. Yan and S. E. Salcudean, "Teleoperation controller design using H_∞ optimization with application to motion-scaling," *IEEE Trans. Control Syst. Technol.*, vol. 4, no. 3, pp. 244–258, May 1996.

[5] D. A. Lawrence, "Stability and transparency in bilateral teleoperation," *IEEE Trans. Robot. Autom.*, vol. 9, no. 5, pp. 624–637, Oct. 1993.

[6] A. Aziminejad, M. Tavakoli, R. V. Patel, and M. Moallem, "Transparent time-delayed bilateral teleoperation using wave variables," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 3, pp. 548–555, May 2008.

[7] P. Malysz and S. Sirouspour, "Nonlinear and filtered force/position mappings in bilateral teleoperation with application to enhanced stiffness discrimination," *IEEE Trans. Robot.*, vol. 25, no. 5, pp. 1134–1149, Oct. 2009.

[8] R. Anderson and M. W. Spong, "Bilateral control of teleoperators with time delay," *IEEE Trans. Autom. Control*, vol. 34, no. 5, pp. 494–501, May 1989.

[9] J.-P. Richard, "Time-delay systems: An overview of some recent advances and open problems," *Automatica*, vol. 39, no. 10, pp. 1667–1694, 2003.

[10] G. Niemeyer and J.-J. E. Slotine, "Telemanipulation with time delays," *Int. J. Robot. Res.*, vol. 23, no. 9, pp. 873–890, 2004.

[11] E. Nuño, L. Basañez, and R. Ortega, "Passivity-based control for bilateral teleoperation: A tutorial," *Automatica*, vol. 47, no. 3, pp. 485–495, Mar. 2011.

[12] G. Niemeyer and J.-J.-E. Slotine, "Stable adaptive teleoperation," *IEEE J. Oceanic Eng.*, vol. 16, no. 1, pp. 152–162, Jan. 1991.

[13] G. D. Niemeyer, "Using wave variables in time delayed force reflecting teleoperation," Ph.D. dissertation, Massachusetts Inst. Technol., Cambridge, MA, USA, 1996.

[14] N. Chopra, M. W. Spong, S. Hirche, and M. Buss, "Bilateral teleoperation over the Internet: The time varying delay," in *Proc. Amer. Control Conf.*, vol. 1, Jun. 2003, pp. 155–160.

[15] K. Kawashima, K. Tadano, G. Sankaranarayanan, and B. Hannaford, "Bilateral teleoperation with time delay using modified wave variables," in *Proc. IEEE/RSJ Int. Intell. Robots Syst.*, Sep. 2008, pp. 424–429.

[16] S. J. Lee and H. S. Ahn, "Controller designs for bilateral teleoperation with input saturation," *Control Eng. Pract.*, vol. 33, pp. 35–47, Dec. 2014.

[17] H. Li and K. Kawashima, "Achieving stable tracking in wave-variable-based teleoperation," *IEEE/ASME Trans. Mechatronics*, vol. 19, no. 5, pp. 1574–1582, Oct. 2014.

[18] J. Guo, C. Liu, and P. Poignet, "Stable and enhanced position-force tracking for bilateral teleoperation with time delay," in *Proc. Eur. Control Conf. (ECC)*, Jul. 2015, pp. 1980–1985.

[19] J.-H. Ryu, D.-S. Kwon, and B. Hannaford, "Stable teleoperation with time-domain passivity control," *IEEE Trans. Robot. Autom.*, vol. 20, no. 2, pp. 365–373, Apr. 2004.

[20] V. Chawda and M. K. O'Malley, "Position synchronization in bilateral teleoperation under time-varying communication delays," *IEEE/ASME Trans. Mechatronics*, vol. 20, no. 1, pp. 245–253, Feb. 2015.

[21] C. Seo, J.-P. Kim, J. Kim, H.-S. Ahn, and J. Ryu, "Robustly stable bilateral teleoperation under time-varying delays and data losses: An energy-bounding approach," *J. Mech. Sci. Technol.*, vol. 25, no. 8, pp. 2089–2100, 2011.

[22] Y. Ye, Y.-J. Pan, and T. Hilliard, "Bilateral teleoperation with time-varying delay: A communication channel passification approach," *IEEE/ASME Trans. Mechatronics*, vol. 18, no. 4, pp. 1431–1434, Aug. 2013.

[23] Y. Ye, Y.-J. Pan, Y. Gupta, and J. Ware, "A power based time domain passivity control for haptic interfaces," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 4, pp. 874–883, Jul. 2011.

[24] Z. Chen, Y. J. Pan, J. Gu, and S. Forbrigger, "A novel multilateral teleoperation scheme with power-based time-domain passivity control," *Trans. Inst. Meas. Control*, vol. 40, no. 11, p. 3252–3262, 2018.

[25] J. A. Esclusa, "Time domain passivity control for delayed teleoperation," Ph.D. dissertation, Univ. Politécnica de Madrid, Madrid, Spain, 2014.

[26] J. Rebelo and A. Schiele, "Time domain passivity controller for 4-channel time-delay bilateral teleoperation," *IEEE Trans. Haptics*, vol. 8, no. 1, pp. 79–89, Jan. 2015.

[27] H. Li and K. Kawashima, "Bilateral teleoperation with delayed force feedback using time domain passivity controller," *Robot. Comput.-Integr. Manuf.*, vol. 37, pp. 188–196, Feb. 2016.

[28] D. Sun, F. Naghdy, and H. Du, "Wave-variable-based passivity control of four-channel nonlinear bilateral teleoperation system under time delays," *IEEE/ASME Trans. Mechatronics*, vol. 21, no. 1, pp. 238–253, Feb. 2016.

[29] D. Lee and M. W. Spong, "Passive bilateral teleoperation with constant time delay," *IEEE Trans. Robot.*, vol. 22, no. 2, pp. 269–281, Apr. 2006.

[30] E. Nuño, R. Ortega, N. Barabanov, and L. Basañez, "A globally stable PD controller for bilateral teleoperators," *IEEE Trans. Robot.*, vol. 24, no. 3, pp. 753–758, Jun. 2008.

[31] E. Nuño, D. Valle, I. Sarras, and L. Basañez, "Bilateral teleoperation of flexible-joint manipulators with dynamic gravity compensation and variable time-delays," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, Nov. 2013, pp. 5439–5444.

[32] E. Nuño, L. Basañez, C. López-Franco, and N. Arana-Daniel, "Stability of nonlinear teleoperators using PD controllers without velocity measurements," *J. Franklin Inst.*, vol. 351, no. 1, pp. 241–258, 2014.

[33] E. Slawiñski and V. Mut, "PD-like controllers for delayed bilateral teleoperation of manipulators robots," *Int. J. Robust Nonlinear Control*, vol. 25, no. 12, pp. 1801–1815, 2015.

[34] C.-C. Hua and X. P. Liu, "Delay-dependent stability criteria of teleoperation systems with asymmetric time-varying delays," *IEEE Trans. Robot.*, vol. 26, no. 5, pp. 925–932, Oct. 2010.

[35] C. Hua and Y. Yang, "Bilateral teleoperation design with/without gravity measurement," *IEEE Trans. Instrum. Meas.*, vol. 61, no. 12, pp. 3136–3146, Dec. 2012.

[36] C. Hua and X. P. Liu, "A new coordinated slave torque feedback control algorithm for network-based teleoperation systems," *IEEE/ASME Trans. Mechatronics*, vol. 18, no. 2, pp. 764–774, Apr. 2013.

[37] C. color red Hua and P. X. Liu, "Teleoperation system design over the Internet with velocity estimation," in *Proc. IEEE Int. Workshop Haptic Audio Vis. Environ. Games*, Nov. 2009, pp. 153–158.

[38] C. Hua and P. X. Liu, "Teleoperation over the Internet with/without velocity signal," *IEEE Trans. Instrum. Meas.*, vol. 60, no. 1, pp. 4–13, Jan. 2011.

[39] D.-H. Zhai and Y. Xia, "Robust saturation-based control of bilateral teleoperation without velocity measurements," *Int. J. Robust Nonlinear Control*, vol. 25, no. 15, pp. 2582–2607, 2015.

[40] F. Hashemzadeh, I. Hassanzadeh, and M. Tavakoli, "Teleoperation in the presence of varying time delays and sandwich linearity in actuators," *Automatica*, vol. 49, no. 9, pp. 2813–2821, 2013.

[41] C. Hua, Y. Yang, and P. X. Liu, "Output-feedback adaptive control of networked teleoperation system with time-varying delay and bounded inputs," *IEEE/ASME Trans. Mechatronics*, vol. 20, no. 5, pp. 2009–2020, Oct. 2015.

[42] F. Hashemzadeh and M. Tavakoli, "Position and force tracking in nonlinear teleoperation systems under varying delays," *Robotica*, vol. 33, pp. 1003–1016, May 2015.

[43] L. Chan, F. Naghdy, and D. Stirling, "Position and force tracking for non-linear haptic telemanipulator under varying delays with an improved extended active observer," *Robot. Auto. Syst.*, vol. 75, pp. 145–160, Jan. 2016.

[44] A. Shahdi and S. Sirouspour, "Improved transparency in bilateral teleoperation with variable time delay," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, Oct. 2009, pp. 4616–4621.

[45] I. Sarras, E. Nuño, and L. Basañez, "An adaptive controller for nonlinear teleoperators with variable time-delays," *J. Franklin Inst.*, vol. 351, no. 10, pp. 4817–4837, 2014.

[46] Y. C. Liu and M. H. Khong, "Adaptive control for nonlinear teleoperators with uncertain kinematics and dynamics," *IEEE/ASME Trans. Mechatronics*, vol. 20, no. 5, pp. 2550–2562, Oct. 2015.
[47] S. Islam, P. X. Liu, A. El Saddik, and Y. B. Yang, "Bilateral control of teleoperation systems with time delay," *IEEE/ASME Trans. Mechatronics*, vol. 20, no. 1, pp. 1–12, Feb. 2015.
[48] K. Abidi, Y. Yildiz, and B. E. Korpe, "Explicit time-delay compensation in teleoperation: An adaptive control approach," *Int. J. Robust Nonlinear Control*, vol. 26, no. 15, pp. 3388–3403, 2016.
[49] M. Laghi, A. Ajoudani, M. Catalano, and A. Bicchi, "Tele-impedance with force feedback under communication time delay," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst. (IROS)*, Sep. 2017, pp. 2564–2571.
[50] A. Ajoudani, N. Tsagarakis, and A. Bicchi, "Tele-impedance: Teleoperation with impedance regulation using a body–machine interface," *Int. J. Robot. Res.*, vol. 31, no. 13, pp. 1642–1656, 2012.
[51] B. Zhang, A. Kruszewski, and J.-P. Richard, "A novel control design for delayed teleoperation based on delay-scheduled Lyapunov–Krasovskii functionals," *Int. J. Control*, vol. 87, no. 8, pp. 1694–1706, 2014.
[52] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*, vol. 3. New York, NY, USA: Wiley, 2006.
[53] E. Nuño, L. Basañez, R. Ortega, and M. Spong, "Position tracking for non-linear teleoperators with variable time delay," *Int. J. Robot. Res.*, vol. 28, no. 7, pp. 895–910, 2009.
[54] I. Karafyllis, P. Pepe, and Z.-P. Jiang, "Input-to-output stability for systems described by retarded functional differential equations," *Eur. J. Control*, vol. 14, no. 6, pp. 539–555, 2008.
[55] I. Karafyllis, P. Pepe, and Z.-P. Jiang, "Global output stability for systems described by retarded functional differential equations: Lyapunov characterizations," *Eur. J. Control*, vol. 14, no. 6, pp. 516–536, 2008.
[56] B. Siciliano and L. Villani, *Robot Force Control*, vol. 540. New York, NY, USA: Springer, 2012.

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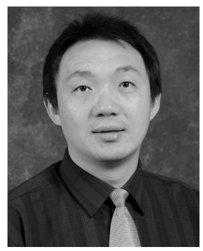

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