Connectivity-Preserving Synchronization of Time-Delay Euler-Lagrange Networks With Bounded Actuation

Article *in* IEEE Transactions on Cybernetics · May 2019

DOI: 10.1109/TCYB.2019.2914403

3 authors:

Yuan Yang
Polytechnique Montréal

Yang Shi
University of Victoria

Daniela Constantinescu
University of Victoria

Connectivity-Preserving Synchronization of Time-Delay Euler–Lagrange Networks With Bounded Actuation

Yuan Yang, Yang Shi, *Fellow, IEEE*, and Daniela Constantinescu, *Member, IEEE*

***Abstract*—This paper proposes a strategy to overcome the threats posed to connectivity-preserving synchronization of Euler–Lagrange networks by time-varying delays and bounded actuation. It first introduces a suitable distributed negative gradient plus damping injection controller, based on which it establishes that the local connectivity of time-delay Euler–Lagrange networks can be preserved by appropriately regulating the interagent connections and increasing the damping injection locally. Yet, actuator saturation may impede the proper modulation of the interagent connections and the injection of sufficient damping and, thus, may threaten the maintenance of connectivity. This paper then develops an indirect coupling control framework which integrates the bounded actuation constraints into the control design. The framework endows every agent with a virtual proxy and couples initially adjacent agents through their virtual proxies. The interproxy couplings then tackle the time-varying delays while the agent-proxy couplings account for the saturation of actuators. Lyapunov–Krasovskii analysis proves that the indirect coupling strategy can drive time-delay Euler–Lagrange networks with bounded actuation to connectivity-preserving synchronization by limiting the energy of the agent-proxy and interproxy couplings according to the actuation constraints. Experiments with Geomagic Touch haptic robots validate the proposed designs compared to a conventional proportional plus damping controller.**

***Index Terms*—Connectivity, Euler–Lagrange systems, saturation, synchronization, time-varying delays.**

I. Introduction

Distributed synchronization of multiagent systems (MASs) drives all agents to the same state using only local and 1-hop state signals [1]. Established strategies employ static proportional (P) control for first-order MASs [2], proportional–derivative (PD) control for second-order MASs [3], and proportional plus damping (P+d) control for Euler–Lagrange (EL) networks [4]. Because practical interagent communications are constrained by distances, the connectivity of multiagent networks may be destroyed under the control of conventional coordination strategies [5].

For MASs with first-order agents, synchronization can be formulated as the minimization of a potential energy that is a function of interagent distances and has a unique minimum at the synchronization configuration. A negative gradient-based controller can then synchronize the MAS. If the potential function is quadratic in interagent distances, the negative gradient law is a form of P control. Distributed P-type controls that guarantee the connectivity and synchronization of kinematic MASs can be derived from unbounded [6]–[9] or bounded [10]–[13] potentials. Nonsmooth gradient-based controllers can achieve finite-time synchronization in the presence of disturbances [14] and Lipschitz nonlinearities [15]. Other distributed gradient-based strategies provide connectivity in the presence of actuator saturation [16] or obstacles [17], [18], strong connectivity in directed graphs in the presence of disturbances [19] or intermittent connectivity [20]–[22]. Recent research investigates the robustness and invariance of connectivity preservation in the presence of additional control terms [23], and the tradeoffs among bounded controls, connectivity maintenance, and additional control objectives [24].

For MASs with second-order, including EL, agents, synchronization can be formulated as the minimization of an energy function with a unique minimum at the synchronization state and with two components: 1) a potential energy function like that of MASs with first-order agents and 2) a kinetic energy function of agent velocities. A negative gradient plus damping injection strategy, like conventional PD and P+d control, can then synchronize the second-order MAS. In the absence of actuation constraints, distributed gradient-based control can also guarantee connectivity-preserving consensus for double integrator MASs [25]. Robust gradient-based control can maintain connectivity during the coordination of double integrators with Lipschitz-like dynamic nonlinearities [26], and during leader–follower coordination of double integrators [27]–[32] and of EL agents [33], [34]. Integral terms added to sliding mode and conventional PD controllers can robustly preserve connectivity during rendezvous [31], flocking [35], [36], and formation tracking [37], [38]. Decentralized algebraic connectivity estimation can preserve global connectivity in cooperative control of multirobots [39]–[42]. A sufficient condition on control forces for preserving connectivity of double integrator MASs has been investigated in [43].

Manuscript received January 29, 2019; accepted April 19, 2019. Date of publication May 21, 2019; date of current version June 23, 2021. This work was supported in part by the CSC Scholarship under Grant 201506120045, and in part by NSERC under Grant 34119 and Grant 34771. This paper was recommended by Associate Editor T. Huang. *(Corresponding author: Daniela Constantinescu.)*

The authors are with the Department of Mechanical Engineering, University of Victoria, Victoria, BC V8W 2Y2, Canada (e-mail: yangyuan@uvic.ca; yshi@uvic.ca; danielac@uvic.ca).

This article has supplementary material provided by the authors and color versions of one or more figures available at https://doi.org/10.1109/TCYB.2019.2914403.

Digital Object Identifier 10.1109/TCYB.2019.2914403

At the communications level, recurrent proximity maintenance [20], [22]; switching graphs [44]; directed graphs [19]; and intermittent algebraic connectivity estimators [42], [45], [46] tackle threats to connectivity due to limited agent communication ranges. Threats due to time-varying communication delays are considered only for attitude synchronization [47], for EL MASs with uncertain parameters [48] and for EL networks without velocity measurements [49]. The threats posed to the connectivity-preserving synchronization of networked EL systems by combined time-varying communication delays and limited actuation are still unclear. This paper proposes a distributed control framework to overcome them.

This paper contributes to research on connectivity-preserving synchronization of time-delay EL networks with bounded actuation as follows.

1) It introduces a potential function from which it derives a gradient plus damping injection control that guarantees connectivity-preserving synchronization of time-delay EL networks with unbounded actuation. Lyapunov stability analysis reveals that the designed control maintains connectivity by modulating the coupling stiffness and by increasing the injected damping. Because actuator saturation restricts the coupling strength and added damping, the strategy is insufficient for connectivity-preserving synchronization of time-delay EL networks with bounded actuation.
2) It develops a coupling framework which, to the authors' best knowledge, is the first strategy to guarantee the connectivity of time-delay EL networks with bounded actuation. The strategy: a) equips each agent with its local second-order virtual proxy; b) couples the agents to their proxies through saturated P+d controls; and c) connects the proxies of initially adjacent agents through properly designed gradient plus damping injection control. Because it couples each pair of adjacent agents indirectly, through their proxies, the proposed strategy is called an indirect coupling framework in this paper. The strategy has three salient features: a) the saturated P+d agent controls exploit their limited actuation more fully, as in the regulation control of a single robot [50]; b) the virtual proxies split the interagent couplings into agent-proxy and interproxy couplings and, thus, permit to tackle time-varying delays and actuator saturation separately, the former in interproxy couplings and the latter in agent-proxy couplings; and c) uniquely to the design, the minimization of the agent-proxy coupling energy on the boundary of a ball limits the interproxy coupling energy appropriately for guaranteed preservation of local connectivity.
3) It compares experimentally the proposed controllers to conventional P+d control [4] to demonstrate that: a) conventional P+d control does not guarantee the local connectivity of time-delay EL networks, and thus may fail to synchronize them; b) given full actuation, the proposed gradient plus damping injection control can drive time-delay EL networks that are initially at rest to connectivity-preserving synchronization; and c) the proposed indirect coupling framework guarantees connectivity-preserving synchronization of time-delay EL networks in the presence of actuator saturation.

II. Preliminaries

Consider a network of N nonredundant EL agents with dynamics

$$\mathbf{M}_i(\mathbf{x}_i)\ddot{\mathbf{x}}_i + \mathbf{C}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i)\dot{\mathbf{x}}_i = \mathbf{u}_i \quad i = 1, \ldots, N. \tag{1}$$

In (1): the subscript i indexes the agent; $\mathbf{x}_i$, $\dot{\mathbf{x}}_i$, and $\ddot{\mathbf{x}}_i$ are its n-dimensional position, velocity, and acceleration; $\mathbf{M}_i(\mathbf{x}_i)$ and $\mathbf{C}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i)$ are its matrices of inertia and of Coriolis and centrifugal effects, respectively; and $\mathbf{u}_i$ is the control force applied to the agent. The dynamics (1) have the following properties [51].

P.1: The inertia matrix $\mathbf{M}_i(\mathbf{x}_i)$ is symmetric, positive definite, and uniformly bounded by $\mathbf{0} \prec \lambda_{i1}\mathbf{I} \preceq \mathbf{M}_i(\mathbf{x}_i) \preceq \lambda_{i2}\mathbf{I} \prec \infty$, with $\lambda_{i2} \geq \lambda_{i1} > 0$.

P.2: The matrix $\dot{\mathbf{M}}_i(\mathbf{x}_i) - 2\mathbf{C}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i)$ is skew-symmetric.

For simplicity of notation, $\mathbf{M}_i$ and $\mathbf{C}_i$ indicate $\mathbf{M}_i(\mathbf{x}_i)$ and $\mathbf{C}_i(\mathbf{x}_i, \dot{\mathbf{x}}_i)$, respectively, in the remainder of this paper.

Further, consider that each agent of the EL network (1) can both send and receive information from other agents that are closer than a distance r from it. Agents i and j are called adjacent, or neighbors, at time t if they exchange information at all time until t. Note that to be adjacent at time t, agents i and j must have been within communication distance r of each other, and thus neighbors, at all time, that is, $\|\mathbf{x}_{ij}(\tau)\| = \|\mathbf{x}_i(\tau) - \mathbf{x}_j(\tau)\| < r$, $\forall \tau \in [0, t]$. The communication graph $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$ of the EL network (1) consists of a set of nodes $\mathcal{V} = \{1, \ldots, N\}$, each associated with one agent in the network, and a set of communication edges $\mathcal{E}(t) = \{(i, j) \in \mathcal{V} \times \mathcal{V} \mid \|\mathbf{x}_{ij}(\tau)\| < r \ \forall \tau \in [0, t]\}$, each associated with a communication link in the network. A path in the graph $\mathcal{G}(t)$ is a sequence of connected edges (i, j), $(j, k), \ldots,$ that belong to $\mathcal{E}(t)$. The graph $\mathcal{G}(t)$ is connected iff there exists a path between each pair of agents.

The connectivity-preserving synchronization of delay-free EL networks has been formulated in [25] as follows.

Problem 1: For the EL network (1), find a distributed control law such that: 1) $\|\mathbf{x}_{ij}(t)\| < r$, $\forall t \geq 0$ and $\forall (i, j) \in \mathcal{E}(0)$ and 2) $\|\mathbf{x}_{ij}(t)\| \to 0$ as $t \to \infty$ $\forall (i, j) \in \mathcal{E}(0)$.

Remark 1: A connected communication graph is sufficient for synchronizing an EL network. In practical applications, in which agents can exchange information only when they are sufficiently close to each other, graph edges and, with them, graph connectivity may be destroyed during control. Then, the synchronization task may fail due to the disconnection of the communication graph. The threat posed to synchronization by a limited agent communication distance r can be overcome by preserving local connectivity during synchronization, that is, by guaranteeing $\mathcal{E}(t) = \mathcal{E}(0)$ $\forall t \geq 0$. In the absence of communication delays, all initial edges of the communication graph are maintained at all times iff all pairs of initially adjacent agents are kept within their communication distance r at all times, that is, iff $\|\mathbf{x}_{ij}(t)\| < r$ $\forall (i, j) \in \mathcal{E}(0)$ and $\forall t \geq 0$. Compared to conventional consensus, the connectivity-preserving synchronization Problem 1 imposes the additional constraint 1) on the interagent distances $\|\mathbf{x}_{ij}(t)\|$.

To preserve the connectivity of the EL network (1) in the presence of time-varying communication delays and bounded actuation, this paper relies on three assumptions.

Assumption 1: The transmission of information from agent i to agent j is affected by the time-varying delay $T_{ij}(t)$, upper bounded by $T_{ij}(t) \leq \overline{T}_{ij}$.

Assumption 2: Initially, each agent i of the EL network (1) is at rest, and has been at rest longer than the maximum transmission delay from it to its neighbors j, that is, $\dot{\mathbf{x}}_i(\tau) = \mathbf{0}\ \forall \tau \in [-\overline{T}_i, 0]$ and $\forall i = 1, \ldots, N$, where $\overline{T}_i = \max_{(i,j)\in\mathcal{E}(0)}(\overline{T}_{ij})$.

Assumption 3: The initial communication graph of the EL network (1) is undirected and connected, and each pair of initially adjacent agents (i, j) is strictly within their communication distance r at $t = 0$, that is, $\|\mathbf{x}_{ij}(0)\| = \|\mathbf{x}_{ji}(0)\| < r - \epsilon$ for some $\epsilon > 0$ if $(i, j) \in \mathcal{E}(0)$.

Remark 2: The upper bounds on the time-varying delays in Assumption 1 determine the damping needed for stability and for preserving the existing communication links in a time-delay EL network. Assumption 2 is inspired by the observation that second-order MASs with limited actuation cannot achieve connectivity-preserving consensus from certain initial states (see Section IV-A [52]). Assumption 3 has been proposed in gradient-based connectivity-preserving consensus control of second-order MASs [25], [29], [36] to account for the inertia of agents with second-order dynamics.

III. Main Results

This section first develops a gradient plus damping control for connectivity-preserving synchronization of time-delay EL networks with unbounded actuation. Then, it integrates the gradient-based control into an indirect coupling framework that provably copes with saturating actuators.

A. Time-Delay EL Network With Unbounded Actuation

Let the following potential function describe the energy stored in each initial link $(i, j) \in \mathcal{E}(0)$ at time t:

$$\psi\left(\|\mathbf{x}_{ij}\|\right) = \frac{P \cdot \|\mathbf{x}_{ij}\|^2}{r^2 - \|\mathbf{x}_{ij}\|^2 + Q} \tag{2}$$

where P and Q are positive constants to be determined, and $\|\mathbf{x}_{ij}\|$ is used in place of $\|\mathbf{x}_{ij}(t)\|$ to simplify notation. Further, let the connectivity-preserving controller of agent i be

$$\mathbf{u}_i = -\sum_{j\in\mathcal{N}_i} \nabla_i \psi\left(\|\mathbf{x}_{ij}^d\|\right) - K_i \dot{\mathbf{x}}_i \tag{3}$$

where $K_i > 0$, $\mathbf{x}_{ij}^d = \mathbf{x}_i - \mathbf{x}_{jd}$ with $\mathbf{x}_{jd} = \mathbf{x}_j(t - T_{ji}(t))$ the delayed data received by agent i from agent j, and

$$\nabla_i \psi\left(\|\mathbf{x}_{ij}^d\|\right) = \frac{2P \cdot \left(r^2 + Q\right)}{\left(r^2 - \left\|\mathbf{x}_{ij}^d\right\|^2 + Q\right)^2} \cdot \left(\mathbf{x}_i - \mathbf{x}_{jd}\right) \tag{4}$$

are the gradients of $\psi(\|\mathbf{x}_{ij}^d\|)$ with respect to $\mathbf{x}_i$.

Remark 3: The potential function $\psi(\|\mathbf{x}_{ij}\|)$ in (2) is continuous, positive definite, and strictly increasing with respect to $\|\mathbf{x}_{ij}\| \in [0, r]$. Using Assumption 3, $\|\mathbf{x}_{ij}(0)\| < r - \epsilon < r$ for every $(i, j) \in \mathcal{E}(0)$, it can be shown that the controller (3) can be designed to guarantee the condition

$$\psi\left(\|\mathbf{x}_{ij}(t)\|\right) < \psi(r) = \frac{Pr^2}{Q} \quad \forall t \geq 0.$$

In turn, this condition implies that agents i and j are within communication distance r of each other at all times in the absence of time-varying transmission delays [25].

Remark 4: For delay-free communications, $\|\mathbf{x}_{ij}(t)\| < r$ for all $t \geq 0$ is a sufficient condition for preserving the undirected communication link between agents i and j during synchronization, that is, for $(i, j) \in \mathcal{E}(0) \Rightarrow (i, j) \in \mathcal{E}(t) \quad \forall t \geq 0$. For delayed communications, at time t, agent i receives the information sent by agent j at time $t - T_{ji}(t)$ and agent j receives the information sent by agent i at time $t - T_{ij}(t)$. The undirected communication link between agents i and j exists at time t, that is, $(i, j) \in \mathcal{E}(t)$, iff $\|\mathbf{x}_i(t) - \mathbf{x}_j(t - T_{ji}(t))\| \leq r$ and $\|\mathbf{x}_j(t) - \mathbf{x}_i(t - T_{ij}(t))\| \leq r$. To account for bounded time-varying communication delays, this paper defines the set of neighbors of agent i at time t as follows:

$$\mathcal{N}_i = \left\{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}(0) \text{ and } |\mathbf{x}_{ij}|_T < r\right\} \tag{5}$$

where

$$|\mathbf{x}_{ij}|_T = \sup_{-\overline{T}_{ji}\leq\tau\leq 0} \left\|\mathbf{x}_i(t) - \mathbf{x}_j(t + \tau)\right\|. \tag{6}$$

Connectivity-preserving synchronization can then be analyzed using the following Lyapunov candidate:

$$V = V_k + V_p + \omega \cdot V_d \tag{7}$$

where $\omega > 0$ and

$$\begin{aligned} V_k &= \frac{1}{2}\sum_{i=1}^{N} \dot{\mathbf{x}}_i^\mathsf{T} \mathbf{M}_i \dot{\mathbf{x}}_i \\ V_p &= \frac{1}{2}\sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i(0)} \psi\left(\|\mathbf{x}_{ij}\|\right) \\ V_d &= \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i(0)} \overline{T}_{ji} \cdot \int_{t-\overline{T}_{ji}}^{t} \left\|\dot{\mathbf{x}}_j(\theta)\right\|^2 d\theta \end{aligned} \tag{8}$$

with $j \in \mathcal{N}_i(0)$ if and only if $(i, j) \in \mathcal{E}(0)$ by Assumptions 2 and 3. In (8), V_k is the kinetic energy of the EL network; V_p is the potential energy of all initial interagent links $(i, j) \in \mathcal{E}(0)$; and V_d measures the impact of time-varying delays on connectivity maintenance.

The analysis is facilitated by the following lemma.

Lemma 1: Let the EL network (1) satisfy Assumptions 1–3, let $\overline{r} = r - \kappa \cdot \epsilon$ with $0 < \kappa < 1$, and select P, Q, and ω by

$$\left[\overline{r}^2 - M \cdot (r - \epsilon)^2\right]Q \geq \left[\overline{r}^2 + M \cdot (r - \overline{r})^2\right](r - \epsilon)^2 - \overline{r}^2 r^2 \tag{9}$$

where $M = |\mathcal{E}(0)|$ is the size of $\mathcal{G}(0)$, and by

$$\omega \geq \frac{P \cdot \bar{r}^2}{\kappa^2 \epsilon^2 \cdot \left(r^2 - \bar{r}^2 + Q\right)}. \quad (10)$$

If $V(\tau) \leq V(0)$ for any $\tau \in [0, t]$, then $|\mathbf{x}_{ij}|_T < r$ and thus $\|\mathbf{x}_{ij}^d\| < r$ for every $(i, j) \in \mathcal{E}(0)$, that is, all initial links are preserved regardless of communication delays.

Proof: Assumption 2 on zero initial velocities of the network implies that $V_k(0) = V_d(0) = 0$ and, thus, $V(0) = V_p(0)$. Choosing Q by (9) leads to

$$V_p(0) < \frac{MP \cdot (r - \epsilon)^2}{r^2 - (r - \epsilon)^2 + Q} \leq \frac{P \cdot \bar{r}^2}{r^2 - \bar{r}^2 + Q}.$$

Because V_k and V_d are non-negative, it follows that $V_p(\tau) \leq V(\tau)$ for every $\tau \in [0, t]$. Hence, if $V(\tau) \leq V(0)$, then

$$V_p(\tau) \leq V(\tau) \leq V(0) = V_p(0) < \frac{P \cdot \bar{r}^2}{r^2 - \bar{r}^2 + Q}. \quad (11)$$

From $\psi(\|\mathbf{x}_{ij}\|)$ continuous, positive definite, and strictly increasing on $\|\mathbf{x}_{ij}\| \in [0, r]$, it follows that:

$$0 \leq \psi\left(\|\mathbf{x}_{ij}(0)\|\right) < \frac{P \cdot (r - \epsilon)^2}{r^2 - (r - \epsilon)^2 + Q} < \frac{P \cdot \bar{r}^2}{r^2 - \bar{r}^2 + Q}$$

by Assumption 3 and the definition of $\psi(\|\mathbf{x}_{ij}\|)$ in (2). Suppose that at time t, the link with maximal length among all initial links is $\|\mathbf{x}_{lm}(t)\| = \bar{r}$. Then it follows that:

$$\psi(\|\mathbf{x}_{lm}(t)\|) = \frac{P \cdot \bar{r}^2}{r^2 - \bar{r}^2 + Q}.$$

Since $\|\mathbf{x}_{ij}(t)\| \leq \|\mathbf{x}_{lm}(t)\| \; \forall (i, j) \in \mathcal{E}(0) - \{(l, m)\}$, it further follows that:

$$0 \leq \psi\left(\|\mathbf{x}_{ij}(t)\|\right) \leq \frac{P \cdot \bar{r}^2}{r^2 - \bar{r}^2 + Q} \quad \forall (i, j) \in \mathcal{E}(0) - \{(l, m)\}.$$

and that

$$V_p(t) \geq \frac{P \cdot \bar{r}^2}{r^2 - \bar{r}^2 + Q}$$

which contradicts (11). Hence, (11) implies that

$$0 \leq \psi\left(\|\mathbf{x}_{ij}(t)\|\right) \leq V_p(t) < \frac{P \cdot \bar{r}^2}{r^2 - \bar{r}^2 + Q} \quad \forall (i, j) \in \mathcal{E}(0)$$

and further that

$$\left\|\mathbf{x}_{ij}(t)\right\| < \bar{r} \quad \forall (i, j) \in \mathcal{E}(0). \quad (12)$$

Application of the Cauchy–Schwarz inequality leads to

$$\begin{aligned}
\int_{t-\overline{T}_{ji}}^{t} \left\|\dot{\mathbf{x}}_j(\theta)\right\|^2 d\theta &= \sum_{k=1}^{n} \int_{t-\overline{T}_{ji}}^{t} \left|\dot{x}_j^k(\theta)\right|^2 d\theta \\
&= \frac{1}{\overline{T}_{ji}} \sum_{k=1}^{n} \int_{t-\overline{T}_{ji}}^{t} \left|\dot{x}_j^k(\theta)\right|^2 d\theta \int_{t-\overline{T}_{ji}}^{t} 1^2 d\theta \\
&\geq \frac{1}{\overline{T}_{ji}} \sum_{k=1}^{n} \left(\int_{t-\overline{T}_{ji}}^{t} \left|\dot{x}_j^k(\theta)\right| d\theta\right)^2 \\
&\geq \frac{1}{\overline{T}_{ji}} \sum_{k=1}^{n} \left(\sup_{-\overline{T}_{ji} \leq \tau \leq 0} x_j^k(t+\tau) - \inf_{-\overline{T}_{ji} \leq \tau \leq 0} x_j^k(t+\tau)\right)^2 \\
&\geq \frac{1}{\overline{T}_{ji}} \cdot \sup_{-\overline{T}_{ji} \leq \tau_1, \tau_2 \leq 0} \left\|\mathbf{x}_j(t+\tau_1) - \mathbf{x}_j(t+\tau_2)\right\|^2 \\
&\geq \frac{1}{\overline{T}_{ji}} \cdot \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \left\|\mathbf{x}_j(t) - \mathbf{x}_j(t+\tau)\right\|^2
\end{aligned}$$

where $\dot{x}_j^k$ is the kth element of $\dot{\mathbf{x}}_j$. Thus, V_d can be lower bounded by

$$V_d(t) \geq \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i(0)} \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \left\|\mathbf{x}_j(t) - \mathbf{x}_j(t+\tau)\right\|^2.$$

By the selection of ω in (10), it follows that:

$$\begin{aligned}
&\sup_{-\overline{T}_{ji} \leq \tau \leq 0} \left\|\mathbf{x}_j(t) - \mathbf{x}_j(t+\tau)\right\| \\
&\leq \sqrt{V_d(t)} \leq \sqrt{\frac{V(t)}{\omega}} \\
&\leq \sqrt{\frac{V(0)}{\omega}} = \sqrt{\frac{V_p(0)}{\omega}} < \sqrt{\frac{1}{\omega} \cdot \frac{P \cdot \bar{r}^2}{r^2 - \bar{r}^2 + Q}} = \kappa \cdot \epsilon. \quad (13)
\end{aligned}$$

Using (6) after adding (12) and (13) leads to

$$\begin{aligned}
|\mathbf{x}_{ij}|_T &= \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \left\|\mathbf{x}_i(t) - \mathbf{x}_j(t+\tau)\right\| \\
&= \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \left\|\mathbf{x}_i(t) - \mathbf{x}_j(t) + \mathbf{x}_j(t) - \mathbf{x}_j(t+\tau)\right\| \\
&\leq \left\|\mathbf{x}_{ij}(t)\right\| + \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \left\|\mathbf{x}_j(t) - \mathbf{x}_j(t+\tau)\right\| \\
&< \bar{r} + \kappa \cdot \epsilon = r \quad \forall (i, j) \in \mathcal{E}(0) \quad (14)
\end{aligned}$$

so all initial links are preserved regardless of communication delays. ■

Remark 5: The above proof of Lemma 1 is in four steps.

1) Given $V(t) \leq V(0)$ and the parameter selection (9), $V_k \geq 0$, $V_d \geq 0$, and $V(0) = V_p(0)$ imply (11).
2) Because $\psi(\|\mathbf{x}_{ij}\|)$ is continuous, positive definite, and strictly increasing on $\|\mathbf{x}_{ij}\| \in [0, r]$, Assumption 3 together with (11) imply (12) by contradiction.
3) Application of the Cauchy–Schwarz inequality bounds V_d and together with (10) and (11) leads to (13).
4) The definition of $|\mathbf{x}_{ij}|_T$ in (6) and the triangle inequality imply (14).

The proof of (12) by contradiction in step 2) has been developed in [25] for mobile agents with delay-free communications. By contrast, the proof in this paper is for time-delay EL networks. As discussed in Remark 4, connectivity preservation in time-delay networks must account for the delay-induced distortions $\mathbf{x}_j(t) - \mathbf{x}_j(t - T_{ji}(t))$. In this paper, the function V_d in the Lyapunov candidate V serves to bound $\mathbf{x}_j(t) - \mathbf{x}_j(t - T_{ji}(t))$ in (13) and $|\mathbf{x}_{ij}|_T$ in (14).

Theorem 1: Let the EL network (1) satisfy Assumptions 1–3. Then, the controls (3) preserve all initial links and synchronize the network if the parameters P, Q, and ω are selected as in (9) and (10), and the gains K_i are chosen to satisfy

$$\overline{K}_i = K_i - \frac{1}{2} \sum_{j \in \mathcal{N}_i(0)} \left((2\omega + \upsilon) \cdot \overline{T}_{ij} + \upsilon \cdot \overline{T}_{ji}\right) > 0 \quad (15)$$

where $\upsilon = 2PQ^{-3} \cdot (r^2 + Q) \cdot (4r^2 + Q)$.

Proof: By property P.2, the derivative of V_k is

$$\begin{aligned}\dot{V}_k &= \frac{1}{2}\sum_{i=1}^{N}\Big(\dot{\mathbf{x}}_i^{\mathsf{T}}\dot{\mathbf{M}}_i\dot{\mathbf{x}}_i + 2\dot{\mathbf{x}}_i^{\mathsf{T}}\mathbf{M}_i\ddot{\mathbf{x}}_i\Big)\\ &= \frac{1}{2}\sum_{i=1}^{N}\dot{\mathbf{x}}_i^{\mathsf{T}}\big(\dot{\mathbf{M}}_i - 2\mathbf{C}_i\big)\dot{\mathbf{x}}_i + \sum_{i=1}^{N}\dot{\mathbf{x}}_i^{\mathsf{T}}\mathbf{u}_i\\ &= -\sum_{i=1}^{N}\dot{\mathbf{x}}_i^{\mathsf{T}}\sum_{j\in\mathcal{N}_i}\nabla_i\psi\Big(\|\mathbf{x}_{ij}^{d}\|\Big) - \sum_{i=1}^{N}K_i\cdot\|\dot{\mathbf{x}}_i\|^2. \end{aligned} \quad (16)$$

The initially undirected network by Assumption 3 implies that $j\in\mathcal{N}_i(0)$ iff $i\in\mathcal{N}_j(0)$, and thus,

$$\begin{aligned}\dot{V}_p &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)}\Big(\dot{\mathbf{x}}_i^{\mathsf{T}}\nabla_i\psi\big(\|\mathbf{x}_{ij}\|\big) + \dot{\mathbf{x}}_j^{\mathsf{T}}\nabla_j\psi\big(\|\mathbf{x}_{ij}\|\big)\Big)\\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)}\dot{\mathbf{x}}_i^{\mathsf{T}}\nabla_i\psi(\|\mathbf{x}_{ij}\|)\\ &\quad + \frac{1}{2}\sum_{j=1}^{N}\sum_{i\in\mathcal{N}_j(0)}\dot{\mathbf{x}}_i^{\mathsf{T}}\nabla_i\psi(\|\mathbf{x}_{ij}\|)\\ &= \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)}\dot{\mathbf{x}}_i^{\mathsf{T}}\nabla_i\psi(\|\mathbf{x}_{ij}\|). \end{aligned} \quad (17)$$

Constant $\overline{T}_{ji}$ then leads to

$$\dot{V}_d = \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)}\overline{T}_{ji}\cdot\Big(\big\|\dot{\mathbf{x}}_j\big\|^2 - \big\|\dot{\mathbf{x}}_j\big(t-\overline{T}_{ji}\big)\big\|^2\Big). \quad (18)$$

Because $\mathcal{N}_i(t) = \mathcal{N}_i(0)$ for all $i = 1,\ldots,N$ at time $t = 0$, $V(\tau) \le V(0)\ \forall\tau\in[0,t]$ implies by Lemma 1 that $\mathcal{N}_i(t) = \mathcal{N}_i(0)$, that is, that all initial links are maintained $\forall\tau\in[0,t]$. Set invariance [53] can then be leveraged to prove connectivity maintenance by induction on time [25]. Let $V(\tau)\le V(0)\ \forall\tau\in[0,t)$ and, thus, $\mathcal{N}_i(\tau) = \mathcal{N}_i(0)\ \forall\tau\in[0,t)$ in (16). It then suffices to prove that $V(t)\le V(0)$ as follows.

After adding (16)–(18), the derivative of V becomes

$$\begin{aligned}\dot{V} &\le \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)}\Big(\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\nabla_i\psi\big(\|\mathbf{x}_{ij}\|\big) - \dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\nabla_i\psi\Big(\Big\|\mathbf{x}_{ij}^{d}\Big\|\Big)\Big)\\ &\quad - \sum_{i=1}^{N}\Bigg(K_i\cdot\|\dot{\mathbf{x}}_i\|^2 - \sum_{j\in\mathcal{N}_i(0)}\omega\overline{T}_{ji}\cdot\|\dot{\mathbf{x}}_j\|^2\Bigg). \end{aligned} \quad (19)$$

By the definition of $\mathcal{N}_i$ in (5), $j\in\mathcal{N}_i \Rightarrow |\mathbf{x}_{ij}|_T < r$ and, thus, $\|\mathbf{x}_{ij}\| \le |\mathbf{x}_{ij}|_T < r$ and $\|\mathbf{x}_{ij}^{d}\| \le |\mathbf{x}_{ij}|_T < r$. It follows that:

$$\begin{aligned}&\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\nabla_i\psi\big(\big\|\mathbf{x}_{ij}\big\|\big) - \dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\nabla_i\psi\big(\big\|\mathbf{x}_{ij}^{d}\big\|\big)\\ &= \frac{2P\cdot\big(r^2+Q\big)}{\big(r^2-\big\|\mathbf{x}_{ij}\big\|^2+Q\big)^2}\cdot\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\big(\mathbf{x}_i-\mathbf{x}_j\big)\\ &\quad - \frac{2P\cdot\big(r^2+Q\big)}{\Big(r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q\Big)^2}\cdot\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\big(\mathbf{x}_i-\mathbf{x}_{jd}\big)\\ &= \frac{2P\cdot\big(r^2+Q\big)\cdot\dot{\mathbf{x}}_i^{\mathsf{T}}\mathbf{x}_{ij}}{\big(r^2-\big\|\mathbf{x}_{ij}\big\|^2+Q\big)^2} - \frac{2P\cdot\big(r^2+Q\big)\cdot\dot{\mathbf{x}}_i^{\mathsf{T}}\mathbf{x}_{ij}}{\Big(r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q\Big)^2}\\ &\quad - \frac{2P\cdot\big(r^2+Q\big)\cdot\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot(\mathbf{x}_j-\mathbf{x}_{jd})}{\Big(r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q\Big)^2}\\ &= \frac{2P\cdot\big(r^2+Q\big)\cdot\dot{\mathbf{x}}_i^{\mathsf{T}}\mathbf{x}_{ij}}{\big(r^2-\big\|\mathbf{x}_{ij}\big\|^2+Q\big)^2\Big(r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q\Big)^2}\\ &\quad\times\Big(\Big(r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q\Big)^2 - \big(r^2-\big\|\mathbf{x}_{ij}\big\|^2+Q\big)^2\Big)\\ &\quad - \frac{2P\cdot\big(r^2+Q\big)\cdot\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot(\mathbf{x}_j-\mathbf{x}_{jd})}{\Big(r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q\Big)^2}\\ &\le \frac{2P\cdot\big(r^2+Q\big)\cdot|\dot{\mathbf{x}}_i^{\mathsf{T}}\mathbf{x}_{ij}|}{\big(r^2-\big\|\mathbf{x}_{ij}\big\|^2+Q\big)(r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q)}\\ &\quad\times\left(\frac{1}{r^2-\big\|\mathbf{x}_{ij}\big\|^2+Q} + \frac{1}{r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q}\right)\\ &\quad\times\Big|\Big(r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q\Big) - \big(r^2-\big\|\mathbf{x}_{ij}\big\|^2+Q\big)\Big|\\ &\quad + 2PQ^{-2}\cdot\big(r^2+Q\big)\cdot|\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\big(\mathbf{x}_j-\mathbf{x}_{jd}\big)|\\ &\le \frac{4PQ^{-1}\cdot\big(r^2+Q\big)\cdot\Big|\big\|\mathbf{x}_{ij}\big\|^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2\Big|\cdot|\dot{\mathbf{x}}_i^{\mathsf{T}}\mathbf{x}_{ij}|}{\big(r^2-\big\|\mathbf{x}_{ij}\big\|^2+Q\big)\Big(r^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2+Q\Big)}\\ &\quad + 2PQ^{-2}\cdot\big(r^2+Q\big)\cdot\Big|\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\big(\mathbf{x}_j-\mathbf{x}_{jd}\big)\Big|\\ &\le 4PQ^{-3}\cdot\big(r^2+Q\big)\cdot\Big|\big\|\mathbf{x}_{ij}\big\|^2-\Big\|\mathbf{x}_{ij}^{d}\Big\|^2\Big|\cdot|\dot{\mathbf{x}}_i^{\mathsf{T}}\mathbf{x}_{ij}|\\ &\quad + 2PQ^{-2}\cdot\big(r^2+Q\big)\cdot|\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\big(\mathbf{x}_j-\mathbf{x}_{jd}\big)|\\ &\le 8PQ^{-3}\cdot\big(r^2+Q\big)\cdot r\cdot\Big\|\mathbf{x}_{ij}-\mathbf{x}_{ij}^{d}\Big\|\cdot|\dot{\mathbf{x}}_i^{\mathsf{T}}\mathbf{x}_{ij}|\\ &\quad + 2PQ^{-2}\cdot\big(r^2+Q\big)\cdot|\dot{\mathbf{x}}_i^{\mathsf{T}}\cdot\big(\mathbf{x}_j-\mathbf{x}_{jd}\big)|\\ &\le \upsilon\cdot\|\dot{\mathbf{x}}_i\|\cdot\big\|\mathbf{x}_j-\mathbf{x}_{jd}\big\|. \end{aligned} \quad (20)$$

After substitution of (20) into (19), $\dot{V}$ is upper bounded by

$$\begin{aligned}\dot{V} &\le \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)}\upsilon\cdot\|\dot{\mathbf{x}}_i\|\cdot\|\mathbf{x}_j-\mathbf{x}_{jd}\|\\ &\quad - \sum_{i=1}^{N}\Bigg(K_i - \sum_{j\in\mathcal{N}_i(0)}\omega\overline{T}_{ij}\Bigg)\cdot\|\dot{\mathbf{x}}_i\|^2. \end{aligned} \quad (21)$$

Remark 6: In (19), the double summation captures the impact of the delay-induced distortions on connectivity-preserving synchronization. Then, (20) shows that the mismatches due to the time-varying delays can be linearized and upper bounded on $\mathcal{N}_i$. As shown below, this property is key to determining how much damping to inject locally to overcome the threat posed by the delayed-induced distortions.

The Cauchy–Schwarz inequality helps to bound the cumulative effect of the distortions as follows:

$$\begin{aligned}
&\int_0^t \|\dot{\mathbf{x}}_i(\theta)\| \cdot \left\|\mathbf{x}_j(\theta) - \mathbf{x}_{jd}(\theta)\right\| d\theta \\
&\le \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \left(\int_0^t \left\|\mathbf{x}_j(\theta) - \mathbf{x}_{jd}(\theta)\right\|^2 d\theta\right)^{\frac{1}{2}} \\
&= \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \left(\int_0^t \left\| \int_{\theta - T_{ji}(\theta)}^{\theta} \dot{\mathbf{x}}_j(\xi) d\xi \right\|^2 d\theta\right)^{\frac{1}{2}} \\
&= \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \left(\int_0^t \sum_{k=1}^{n} \left| \int_{\theta - T_{ji}(\theta)}^{\theta} \dot{x}_j^k(\xi) d\xi \right|^2 d\theta\right)^{\frac{1}{2}} \\
&\le \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \left(\int_0^t \sum_{k=1}^{n} \overline{T}_{ji} \int_{\theta - T_{ji}(\theta)}^{\theta} |\dot{x}_j^k(\xi)|^2 d\xi d\theta\right)^{\frac{1}{2}} \\
&= \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \left(\overline{T}_{ji} \int_0^t \int_{-T_{ji}(\theta)}^{0} \|\dot{\mathbf{x}}_j(\theta + \tau)\|^2 d\tau d\theta\right)^{\frac{1}{2}} \\
&\le \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \left(\overline{T}_{ji} \int_0^t \int_{-\overline{T}_{ji}}^{0} \|\dot{\mathbf{x}}_j(\theta + \tau)\|^2 d\tau d\theta\right)^{\frac{1}{2}} \\
&= \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \left(\overline{T}_{ji} \int_{-\overline{T}_{ji}}^{0} \int_0^t \|\dot{\mathbf{x}}_j(\theta + \tau)\|^2 d\theta d\tau\right)^{\frac{1}{2}} \\
&\le \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \left(\overline{T}_{ji} \int_{-\overline{T}_{ji}}^{0} \int_0^t \|\dot{\mathbf{x}}_j(\theta)\|^2 d\theta d\tau\right)^{\frac{1}{2}} \\
&= \overline{T}_{ji} \cdot \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \left(\int_0^t \|\dot{\mathbf{x}}_j(\theta)\|^2 d\theta\right)^{\frac{1}{2}} \\
&\le \frac{\overline{T}_{ji}}{2} \cdot \left(\int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta + \int_0^t \|\dot{\mathbf{x}}_j(\theta)\|^2 d\theta\right)
\end{aligned} \tag{22}$$

where $\dot{x}_j^k(\xi)$ is the kth element of $\dot{\mathbf{x}}_j(\xi)$ and Assumption 2 has been used, that is, $\dot{\mathbf{x}}_i(\theta) = \mathbf{0}$ for $\theta \in [-\overline{T}_{ji}, 0]$. Then, time integration of $\dot{V}$ from 0 to t leads to

$$\begin{aligned}
V(t) \le& \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i(0)} \frac{\upsilon \overline{T}_{ji}}{2} \int_0^t \left(\|\dot{\mathbf{x}}_i(\theta)\|^2 + \|\dot{\mathbf{x}}_j(\theta)\|^2\right) d\theta \\
&- \sum_{i=1}^{N} \left(K_i - \sum_{j \in \mathcal{N}_i(0)} \omega \overline{T}_{ij}\right) \int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta + V(0) \\
=& V(0) - \sum_{i=1}^{N} \overline{K}_i \cdot \int_0^t \|\dot{\mathbf{x}}_i(\theta)\|^2 d\theta.
\end{aligned} \tag{23}$$

The selection of $\overline{K}_i > 0$ in (15) guarantees that: 1) $V(t) \le V(0)$ $\forall t \ge 0$ and 2) $\dot{\mathbf{x}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ for $i = 1, \ldots, N$.

Because parameters P, Q, and ω satisfy (9) and (10), Lemma 1 together with $V(t) \le V(0)$ $\forall t \ge 0$ prove that all initial links are preserved, that is, $\mathcal{N}_i(t) = \mathcal{N}_i(0)$ for all $t \ge 0$ and $i = 1, \ldots, N$. By Barbalat's lemma, $\dot{\mathbf{x}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ for $i = 1, \ldots, N$ implies synchronization of the EL network [4], [25]. ■

Remark 7: The proof of Theorem 1 above is by making $\mathcal{N}_i(\tau)$ forward invariant [53]: assume $\mathcal{N}_i(t^-) = \mathcal{N}_i(0)$ and $V(\tau) \le V(0)$ for all $\tau \in [0, t)$ and obtain $V(t) \le V(0)$ which, in turn, implies that $\mathcal{N}_i(t) = \mathcal{N}_i(0)$. The connectivity-preserving synchronization controllers (3) can be designed as follows:

1) select Q to satisfy (9);
2) set ω by (10) after choosing P heuristically;
3) inject sufficient local damping according to (15).

Practical EL networks have limited actuation and the controls (3) may saturate it and become invalid. The following section develops an indirect coupling control framework to integrate the constraints due to bounded actuation into the controller design.

B. Time-Delay EL Network With Bounded Actuation

The indirect coupling framework proposed in this section: 1) endows each agent with a second-order virtual proxy; 2) interconnects the proxies of initially adjacent agents; and 3) couples the agents to their proxies. As shown below, the intervening dynamics of the virtual proxies decouple the time-varying delays from actuator saturation because they permit to tackle the delays in the interproxy couplings and the actuation bounds in the agent-proxy couplings.

The designed dynamics of the virtual proxy of agent i are

$$\hat{\mathbf{M}}_i \ddot{\hat{\mathbf{x}}}_i = \text{Sat}_i\left(P_i \tilde{\mathbf{x}}_i\right) - \sum_{j \in \mathcal{N}_i} \nabla_i \hat{\psi}\left(\left\|\hat{\mathbf{x}}_{ij}^d\right\|\right) - \hat{K}_i \dot{\hat{\mathbf{x}}}_i \tag{24}$$

where $\tilde{\mathbf{x}}_i = \mathbf{x}_i - \hat{\mathbf{x}}_i$, $\hat{\mathbf{x}}_{ij}^d = \hat{\mathbf{x}}_i - \hat{\mathbf{x}}_{jd}$ with $\hat{\mathbf{x}}_{jd} = \hat{\mathbf{x}}_j(t - T_{ji}(t))$, $\text{Sat}_i(\cdot)$ is a vector-valued saturation function with bounds to be determined in Remark 8, $\hat{\mathbf{M}}_i$ is a tunable positive diagonal matrix, P_i and $\hat{K}_i$ are positive constants, $\mathcal{N}_i$ is to be defined in (29), and the potential function $\hat{\psi}(\|\hat{\mathbf{x}}_{ij}\|)$ is defined by

$$\hat{\psi}(\|\hat{\mathbf{x}}_{ij}\|) = \frac{\hat{P} \cdot \|\hat{\mathbf{x}}_{ij}\|^2}{\hat{r}^2 - \|\hat{\mathbf{x}}_{ij}\|^2 + \hat{Q}} \tag{25}$$

with $\hat{r} = r - 2\epsilon/3$, and $\hat{P}$ and $\hat{Q}$ are positive constants to be determined. The gradient of $\hat{\psi}(\|\hat{\mathbf{x}}_{ij}^d\|)$ with respect to $\hat{\mathbf{x}}_i$

$$\nabla_i \hat{\psi}\left(\left\|\hat{\mathbf{x}}_{ij}^d\right\|\right) = \frac{2\hat{P} \cdot \left(\hat{r}^2 + \hat{Q}\right)}{\left(\hat{r}^2 - \left\|\hat{\mathbf{x}}_{ij}^d\right\|^2 + \hat{Q}\right)^2} \cdot \left(\hat{\mathbf{x}}_i - \hat{\mathbf{x}}_{jd}\right) \tag{26}$$

provides the coupling between the proxies of the initially adjacent agents i and j. The nominal control

$$\hat{\mathbf{u}}_i = -P_i \tilde{\mathbf{x}}_i - K_i \dot{\mathbf{x}}_i \tag{27}$$

where K_i is a non-negative constant, connects each EL agent i to its virtual proxy, and completes the design of the indirect coupling framework.

Remark 8: Let the actuation bound of each agent i be $\overline{\boldsymbol{u}}_i = [\bar{u}_i^1 \; \cdots \; \bar{u}_i^n]^\mathsf{T}$. The actual control input $\mathbf{u}_i$ is then a saturated version of the nominal control $\hat{\mathbf{u}}_i$, generally modeled by [50]

$$\mathbf{u}_i = \mathrm{Sat}_i(\hat{\mathbf{u}}_i) = \left[\mathrm{sat}_i^1\left(\hat{u}_i^1\right) \ \cdots \ \mathrm{sat}_i^n\left(\hat{u}_i^n\right)\right]^{\mathsf{T}}. \tag{28}$$

In (28), $\mathrm{Sat}_i(\cdot)$ saturates $\hat{\mathbf{u}}_i$ component-wise by standard saturation functions $\mathrm{sat}_i^k(\cdot)$ with bounds $\overline{u}_i^k$, $k = 1, \ldots, n$. Because the proportional gains P_i and damping gains K_i can be tuned freely, independent of the time-varying delays and actuator saturation, the controller (27) can exploit the available bounded actuation more fully, as discussed in [50], [54], and [55].

Remark 9: The virtual proxy designed in (24) can be regarded as a virtual inertia $\hat{\mathbf{M}}_i$ with position $\hat{\mathbf{x}}_i$ and forced by the right-hand side of (24). The saturated proportional term, $\mathrm{Sat}_i(P_i\tilde{\mathbf{x}}_i)$ in (24) connects the proxy to its EL agent i passively. The gradient term $-\nabla_i\hat{\psi}(\|\hat{\mathbf{x}}_{ij}^d\|)$ connects the proxies of the adjacent agents i and j via delayed communications. Because every two neighboring agents i and j are connected to their proxies and the proxies, in turn, are connected to each other, adjacent agents are indirectly coupled through their proxies.

For delay-free communications, connectivity preservation requires that $\|\mathbf{x}_{ij}(t)\| < r$, $\forall(i,j) \in \mathcal{E}(0)$ and $\forall t \geq 0$. By the triangle inequality, $\|\mathbf{x}_{ij}\| \leq \|\tilde{\mathbf{x}}_i\| + \|\hat{\mathbf{x}}_{ij}\| + \|\tilde{\mathbf{x}}_j\|$ and it suffices to guarantee that

$$\begin{aligned} \left\|\hat{\mathbf{x}}_{ij}(t)\right\| &< \hat{r}, \quad \forall(i,j) \in \mathcal{E}(0) \\ \left\|\tilde{\mathbf{x}}_i(t)\right\| &\leq \frac{\epsilon}{3}, \quad \forall i = 1, \ldots, N. \end{aligned}$$

For delayed communications, as discussed in Remark 4, $\|\hat{\mathbf{x}}_{ij}(t)\| < \hat{r}$ does not guarantee that the proxy of agent i can receive at time t the information sent by the proxy of agent j at time $t - T_{ji}(t)$. Thus, $\|\hat{\mathbf{x}}_{ij}(t)\| < \hat{r}$ is not sufficient for connectivity preservation. To account for the delay-induced mismatches $\|\hat{\mathbf{x}}_j(t) - \hat{\mathbf{x}}_j(t - T_{ji}(t))\|$, the set of neighbors of agent i is redefined by

$$\mathcal{N}_i = \left\{j \in \mathcal{V} \mid (i,j) \in \mathcal{E}(0) \text{ and } |\hat{\mathbf{x}}_{ij}|_T < \hat{r}\right\} \tag{29}$$

where

$$|\hat{\mathbf{x}}_{ij}|_T = \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \left\|\hat{\mathbf{x}}_i(t) - \hat{\mathbf{x}}_j(t+\tau)\right\|. \tag{30}$$

Assumption 3 implies that $\|\mathbf{x}_{ij}(0)\| < r - \epsilon < \hat{r}$, $\forall(i,j) \in \mathcal{E}(0)$. Letting the initial state of the virtual proxy (24) be

$$\hat{\mathbf{x}}_i(\tau) = \mathbf{x}_i(\tau) \quad \text{and} \quad \dot{\hat{\mathbf{x}}}_i(\tau) = \mathbf{0} \quad \forall\tau \in [-\overline{T}_i, 0] \tag{31}$$

ensures that $j \in \mathcal{N}_i(0)$ if and only if $(i,j) \in \mathcal{E}(0)$. Then, the connectivity of the time-delay EL network with bounded actuation is preserved if the set $\mathcal{N}_i$ is rendered invariant, that is,

$$\mathcal{N}_i(t) = \mathcal{N}_i(0) \quad \forall t \geq 0 \tag{32}$$

and if

$$\|\tilde{\mathbf{x}}_i(t)\| \leq \frac{\epsilon}{3} \quad \forall i = 1, \ldots, N, \forall t \geq 0. \tag{33}$$

More specifically, as discussed in Remark 4, the undirected communication link between agents i and j exists at time t, that is, $(i,j) \in \mathcal{E}(t)$, iff $\|\mathbf{x}_i(t) - \mathbf{x}_j(t - T_{ji}(t))\| \leq r$ and $\|\mathbf{x}_j(t) - \mathbf{x}_i(t - T_{ij}(t))\| \leq r$. The conditions (32) and (33) lead to

$$\begin{aligned} |\mathbf{x}_{ij}|_T &= \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \|\mathbf{x}_i(t) - \mathbf{x}_j(t+\tau)\| \\ &= \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \Big(\big\|\mathbf{x}_i(t) - \hat{\mathbf{x}}_i(t) + \hat{\mathbf{x}}_i(t) - \hat{\mathbf{x}}_j(t+\tau) \\ &\qquad + \hat{\mathbf{x}}_j(t+\tau) - \mathbf{x}_j(t+\tau)\big\|\Big) \\ &\leq \left\|\tilde{\mathbf{x}}_i(t)\right\| + \sup_{-\overline{T}_{ji} \leq r \leq 0} \left\|\hat{\mathbf{x}}_i(t) - \hat{\mathbf{x}}_j(t+\tau)\right\| \\ &\quad + \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \|\tilde{\mathbf{x}}_j(t+\tau)\| \\ &= \|\tilde{\mathbf{x}}_i(t)\| + |\hat{\mathbf{x}}_{ij}|_T + \sup_{-\overline{T}_{ji} \leq \tau \leq 0} \|\tilde{\mathbf{x}}_j(t+\tau)\| \\ &< \frac{\epsilon}{3} + \hat{r} + \frac{\epsilon}{3} = r \quad \forall(i,j) \in \mathcal{E}(0) \end{aligned} \tag{34}$$

and thus are sufficient to maintain the connectivity of the time-delay EL network with bounded actuation.

By the design of the virtual proxies (24), actuator saturation makes the agent-proxy couplings dynamic but symmetric and thus passive. The potential energy stored in the coupling between agent i and its proxy is

$$\phi_i(\tilde{\mathbf{x}}_i) = \int_{\mathbf{0}}^{\tilde{\mathbf{x}}_i} \mathrm{Sat}_i(P_i\boldsymbol{\sigma})^{\mathsf{T}} d\boldsymbol{\sigma}. \tag{35}$$

Then, the following three propositions combined give an optimal ϕ^* such that $\|\tilde{\mathbf{x}}_i(t)\| \leq (\epsilon/3)$ is guaranteed if $\phi_i(\tilde{\mathbf{x}}_i) \leq \phi^*$.

Proposition 1: The potential function $\phi_i(\tilde{\mathbf{x}}_i)$ in (35) is convex with respect to $\tilde{\mathbf{x}}_i$ on $\mathbb{R}^n$.

Proof: The gradient of $\phi_i(\tilde{\mathbf{x}}_i)$ with respect to $\tilde{\mathbf{x}}_i$ is

$$\nabla\phi_i(\tilde{\mathbf{x}}_i) = \mathrm{Sat}_i(P_i\tilde{\mathbf{x}}_i).$$

Let $\mathbf{y} \in \mathbb{R}^n$ and $\mathbf{z} \in \mathbb{R}^n$. Then, it holds that

$$\begin{aligned} &(\nabla\phi_i(\mathbf{y}) - \nabla_i\phi(\mathbf{z}))^{\mathsf{T}}(\mathbf{y} - \mathbf{z}) \\ &\quad = (\mathrm{Sat}_i(P_i\mathbf{y}) - \mathrm{Sat}_i(P_i\mathbf{z}))^{\mathsf{T}}(\mathbf{y} - \mathbf{z}) \\ &\quad = \sum_{k=1}^{n}\left(\mathrm{sat}_i^k\left(P_i y^k\right) - \mathrm{sat}_i^k\left(P_i z^k\right)\right)\left(y^k - z^k\right) \geq 0 \end{aligned}$$

because $\mathrm{sat}_i^k(\cdot)$ are nondecreasing functions. By the first-order convexity condition, $\phi_i(\tilde{\mathbf{x}}_i)$ is convex on $\mathbb{R}^n$. ■

Proposition 2: On the ball $B(\mathbf{0}, (\epsilon/3)) = \{\tilde{\mathbf{x}}_i \mid \|\tilde{\mathbf{x}}_i\| \leq (\epsilon/3)\}$, the potential function $\phi_i(\tilde{\mathbf{x}}_i)$ is maximum on the boundary $\|\tilde{\mathbf{x}}_i\| = (\epsilon/3)$ and minimum at the origin $\tilde{\mathbf{x}}_i = \mathbf{0}$.

Proof: The function $\phi_i(\tilde{\mathbf{x}}_i)$ is continuous on the closed and bounded ball $B(\mathbf{0}, (\epsilon/3)) \subseteq \mathbb{R}^n$. Therefore, by the Weierstrass theorem, $\phi_i(\tilde{\mathbf{x}}_i)$ attains its global minimum and maximum on $B(\mathbf{0}, (\epsilon/3))$. Further, $\phi_i(\tilde{\mathbf{x}}_i)$ is convex on $\mathbb{R}^n$. Hence, on the ball $B(\mathbf{0}, (\epsilon/3))$, $\phi_i(\tilde{\mathbf{x}}_i)$ attains its global maximum on the boundary of $B(\mathbf{0}, (\epsilon/3))$, and its global minimum at the point $\nabla\phi_i(\tilde{\mathbf{x}}_i) = \mathrm{Sat}_i(P_i\tilde{\mathbf{x}}_i) = \mathbf{0}$, that is at $\tilde{\mathbf{x}}_i = \mathbf{0}$. ■

Proposition 3: Let ϕ_i^* be the minimum value of ϕ_i on the boundary of the ball $B(\mathbf{0}, (\epsilon/3))$

$$\phi_i^* = \min_{\tilde{\mathbf{x}}_i} \ \phi_i\big(\tilde{\mathbf{x}}_i\big) = \int_{\mathbf{0}}^{\tilde{\mathbf{x}}_i} \text{Sat}_i(P_i \boldsymbol{\sigma})^\top d\boldsymbol{\sigma} \quad \text{s.t.} \ \|\tilde{\mathbf{x}}_i\| = \frac{\epsilon}{3}. \tag{36}$$

If $\phi_i(\tilde{\mathbf{x}}_i) \leq \phi_i^*$, then $\tilde{\mathbf{x}}_i \in B(\mathbf{0}, (\epsilon/3))$.

Proof: Suppose there exists $\tilde{\mathbf{x}}_i \notin B(\mathbf{0}, (\epsilon/3))$ such that $\phi_i(\tilde{\mathbf{x}}_i) \leq \phi_i^*$. Then, there exists $0 < \lambda < 1$ such that $\mathbf{x} = \lambda \mathbf{0} + (1-\lambda)\tilde{\mathbf{x}}_i$ is on the boundary of $B(\mathbf{0}, (\epsilon/3))$, that is, $\|\mathbf{x}\| = (\epsilon/3)$. By the convexity of $\phi_i(\cdot)$, it follows that:

$$\begin{aligned}\phi_i(\mathbf{x}) &= \phi_i\big(\lambda\mathbf{0} + (1-\lambda)\tilde{\mathbf{x}}_i\big) \leq \lambda\phi_i(\mathbf{0}) + (1-\lambda)\phi_i\big(\tilde{\mathbf{x}}_i\big) \\ &= (1-\lambda)\phi_i\big(\tilde{\mathbf{x}}_i\big) < \phi_i^*\end{aligned}$$

which contradicts $\phi_i(\mathbf{x}) \geq \phi_i^*$. Therefore, $\phi_i(\tilde{\mathbf{x}}_i) \leq \phi_i^*$ implies that $\tilde{\mathbf{x}}_i \in B(\mathbf{0}, (\epsilon/3))$. ■

The Lyapunov candidate used to investigate connectivity-preserving synchronization of the time-delay EL network (1) with limited actuation is

$$V = V_k + V_p + \hat{\omega} \cdot V_d \tag{37}$$

where $\hat{\omega} > 0$ and

$$\begin{aligned}V_k &= \frac{1}{2}\sum_{i=1}^{N}\Big(\dot{\mathbf{x}}_i^\top \mathbf{M}_i \dot{\mathbf{x}}_i + \dot{\hat{\mathbf{x}}}_i^\top \hat{\mathbf{M}}_i \dot{\hat{\mathbf{x}}}_i\Big) \\ V_p &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)} \hat{\psi}(\|\hat{\mathbf{x}}_{ij}\|) + \sum_{i=1}^{N}\phi_i(\tilde{\mathbf{x}}_i) \\ V_d &= \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)} \overline{T}_{ji} \cdot \int_{t-\overline{T}_{ji}}^{t} \|\dot{\hat{\mathbf{x}}}_j(\theta)\|^2 d\theta.\end{aligned} \tag{38}$$

Compared to (8), in (38), V_k is the kinetic energy of the EL network plus all the proxies; V_p is the potential energy of all agent-proxy and interproxy couplings; and V_d evaluates the impact of the time-varying delays on the constraints imposed on interproxy distances by connectivity preservation.

Then, the following lemma facilitates the proof of connectivity maintenance.

Lemma 2: Let the EL network (1) satisfy Assumptions 1–3, and let $\tilde{r} = \hat{r} - \kappa \cdot \epsilon/3$ with $0 < \kappa < 1$, $M = |\mathcal{E}(0)|$, and

$$\phi^* = \min_{i=1,\dots,N}\big(\phi_i^*\big).$$

Select $\hat{P}$, $\hat{Q}$, and $\hat{\omega}$ by

$$\Big[\tilde{r}^2 - M\cdot(r-\epsilon)^2\Big]\hat{Q} \geq \Big[\tilde{r}^2 + M\cdot\Big(\hat{r}^2 - \tilde{r}^2\Big)\Big](r-\epsilon)^2 - \hat{r}^2\tilde{r}^2 \tag{39}$$

$$\hat{P} = \frac{\phi^*}{\tilde{r}^2}\cdot\Big(\hat{r}^2 - \tilde{r}^2 + \hat{Q}\Big) \tag{40}$$

$$\hat{\omega} \geq \frac{9\phi^*}{\kappa^2\epsilon^2}. \tag{41}$$

Then, $V(\tau) \leq V(0)\ \forall\tau \in [0, t]$ implies (32) and (33), that is, $\mathcal{N}_i(t) = \mathcal{N}_i(0)$ and $\|\tilde{\mathbf{x}}_i(t)\| \leq (\epsilon/3)\ \forall i = 1, \dots, N$ and $\forall t \geq 0$. By (34), it further implies that $|\mathbf{x}_{ij}|_T < r\ \forall (i,j) \in \mathcal{E}(0)$, that is, all initial links are maintained regardless of the delays.

Proof: By Assumption 2 and the initial state set in (31) of every virtual proxy, it follows that $V_k(0) = V_d(0) = 0$ and $\phi_i(\tilde{\mathbf{x}}_i(0)) = 0$ for $i = 1, \dots, N$. Then, $\hat{Q}$ in (39) and $\hat{P}$ in (40) lead to

$$\begin{aligned}V(0) &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)} \hat{\psi}\big(\|\hat{\mathbf{x}}_{ij}(0)\|\big) \\ &< \frac{M\hat{P}\cdot(r-\epsilon)^2}{\hat{r}^2 - (r-\epsilon)^2 + \hat{Q}} \leq \frac{\hat{P}\cdot\tilde{r}^2}{\hat{r}^2 - \tilde{r}^2 + \hat{Q}} = \phi^*.\end{aligned}$$

If $V(t) \leq V(0)$ for any $t \geq 0$, then $V_k(t) \geq 0$ and $V_d \geq 0$ imply $V_p(t) < \phi^*$. Further, (35) implies $\phi_i(\tilde{\mathbf{x}}_i) \geq 0$ and, thus, V_p defined in (38) implies

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)} \hat{\psi}\big(\|\hat{\mathbf{x}}_{ij}\|\big) < \phi^* = \frac{\hat{P}\cdot\tilde{r}^2}{\hat{r}^2 - \tilde{r}^2 + \hat{Q}}.$$

An analysis by contradiction similar to the analysis in Lemma 1 leads to

$$0 \leq \hat{\psi}\big(\|\hat{\mathbf{x}}_{ij}(t)\|\big) = \frac{\hat{P}\cdot\|\hat{\mathbf{x}}_{ij}(t)\|^2}{\hat{r}^2 - \|\hat{\mathbf{x}}_{ij}(t)\|^2 + \hat{Q}} < \phi^*$$

and thus to

$$\|\hat{\mathbf{x}}_{ij}(t)\| < \tilde{r}, \quad \forall (i,j) \in \mathcal{E}(0). \tag{42}$$

Because $\hat{\psi}(\|\hat{\mathbf{x}}_{ij}(t)\|) \geq 0$, $V_p(t) < \phi^*$ implies that $\phi_i(\tilde{\mathbf{x}}_i) < \phi^*$ and, by Proposition 3, that

$$\|\tilde{\mathbf{x}}_i(t)\| < \frac{\epsilon}{3}, \quad \forall i = 1, \dots, N. \tag{43}$$

For $\hat{\omega}$ selected by (41), $\hat{\psi}(\|\hat{\mathbf{x}}_{ij}(t)\|) \geq 0$, $\phi_i(\tilde{\mathbf{x}}_i) > 0$ and $V(t) \leq V(0) = \phi^*$ lead to

$$V_d(t) \leq \frac{\phi^*}{\hat{\omega}} \leq \frac{\kappa^2\epsilon^2}{9}.$$

Similar to the proof of Lemma 1, the Cauchy–Schwarz inequality can be applied to lower bound V_d by

$$V_d(t) \geq \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)} \sup_{-\overline{T}_{ji}\leq\tau\leq 0} \big\|\hat{\mathbf{x}}_j(t) - \hat{\mathbf{x}}_j(t+\tau)\big\|^2.$$

Hence, it follows that:

$$\sup_{-\overline{T}_{ji}\leq\tau\leq 0} \big\|\hat{\mathbf{x}}_j(t) - \hat{\mathbf{x}}_j(t+\tau)\big\| \leq \sqrt{V_d(t)} \leq \frac{\kappa\cdot\epsilon}{3}. \tag{44}$$

Together, (42) and (44) imply that

$$\begin{aligned}\big|\hat{\mathbf{x}}_{ij}(t)\big|_T &= \sup_{-\overline{T}_{ji}\leq\tau\leq 0} \big\|\hat{\mathbf{x}}_i(t) - \hat{\mathbf{x}}_j(t+\tau)\big\| \\ &= \sup_{-\overline{T}_{ji}\leq\tau\leq 0} \big\|\hat{\mathbf{x}}_i(t) - \hat{\mathbf{x}}_j(t) + \hat{\mathbf{x}}_j(t) - \hat{\mathbf{x}}_j(t+\tau)\big\| \\ &\leq \big\|\hat{\mathbf{x}}_{ij}(t)\big\| + \sup_{-\overline{T}_{ji}\leq\tau\leq 0} \big\|\hat{\mathbf{x}}_j(t) - \hat{\mathbf{x}}_j(t+\tau)\big\| \\ &< \tilde{r} + \frac{\kappa\cdot\epsilon}{3} = \hat{r}\end{aligned} \tag{45}$$

and thus that $\|\hat{\mathbf{x}}_{ij}^d\| < \hat{r}$ for any $(i,j) \in \mathcal{E}(0)$, that is, $\mathcal{N}_i(t) = \mathcal{N}_i(0)$ for all $i = 1, \ldots, N$ and $t \geq 0$.

By (34), (43) together with (45) prove that all initial links are maintained regardless of the delays. ∎

The following proposition facilitates the proof of connectivity-preserving synchronization in Theorem 2.

Proposition 4: Let Sat(·) be a standard saturation function. For any $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{y} \in \mathbb{R}^n$, the following inequality holds:

$$-\mathbf{x}^\mathsf{T}\mathrm{Sat}(\mathbf{x}+\mathbf{y}) \leq -\mathbf{x}^\mathsf{T}\mathrm{Sat}(\mathbf{y}).$$

Proof: It suffices to show that $-x_k\mathrm{sat}(x_k+y_k) \leq -x_k\mathrm{sat}(y_k)$ for any $k = 1, \ldots, n$, where x_k and y_k are the kth elements of $\mathbf{x}$ and $\mathbf{y}$, respectively. Let the bound on sat(·) be s.

1) Let $\mathrm{sat}(x_k+y_k) = x_k + y_k$. If $\mathrm{sign}(x_k) = \mathrm{sign}(y_k)$, then it holds that $-x_k\mathrm{sat}(x_k+y_k) = -x_k^2 - x_k y_k \leq -x_k y_k \leq -x_k\mathrm{sat}(y_k)$. If $\mathrm{sign}(x_k) \neq \mathrm{sign}(y_k)$, then it holds that $-x_k\mathrm{sat}(x_k+y_k) = -x_k^2 + |x_k y_k| \leq |x_k|s$. Further:
   a) if $\mathrm{sat}(y_k) = y_k$, then $-x_k\mathrm{sat}(y_k) = |x_k y_k|$ follows; or
   b) if $\mathrm{sat}(y_k) = s \cdot \mathrm{sign}(y_k)$, then $-x_k\mathrm{sat}(y_k) = |x_k|s$ follows.

   In both cases, $-x_k\mathrm{sat}(x_k+y_k) \leq -x_k\mathrm{sat}(y_k)$ holds.
2) Let $\mathrm{sat}(x_k+y_k) \neq x_k + y_k$. If $\mathrm{sign}(x_k+y_k) = \mathrm{sign}(x_k)$, then it holds that $-x_k\mathrm{sat}(x_k+y_k) = -|x_k|s \leq -x_k\mathrm{sat}(y_k)$. This gives $-x_k\mathrm{sat}(x_k+y_k) \leq -x_k\mathrm{sat}(y_k)$. If $\mathrm{sign}(x_k+y_k) \neq \mathrm{sign}(x_k)$, then $\mathrm{sign}(y_k) \neq \mathrm{sign}(x_k)$ and $|y_k| \geq s + |x_k|$. It follows that $-x_k\mathrm{sat}(x_k+y_k) = |x_k|s$ and $-x_k\mathrm{sat}(y_k) = -x_k \cdot s \cdot \mathrm{sign}(y_k) = |x_k|s$. Hence, $-x_k\mathrm{sat}(x_k+y_k) \leq -x_k\mathrm{sat}(y_k)$. ∎

Theorem 2: Let the time-delay EL network with bounded actuation (1) satisfy Assumptions 1–3. Then, the controls (24) and (27) preserve all initial links and synchronize the network if the parameters $\hat{P}$, $\hat{Q}$, and $\hat{\omega}$ are selected by (39)–(41), and the gains P_i, K_i, and $\hat{K}_i$ satisfy

$$\tilde{K}_i = \hat{K}_i - \frac{1}{2}\sum_{j\in\mathcal{N}_i(0)}\left[\left(2\hat{\omega}+\hat{\upsilon}\right)\cdot\overline{T}_{ij} + \hat{\upsilon}\cdot\overline{T}_{ji}\right] > 0 \tag{46}$$

where $\hat{\upsilon} = 2\hat{P}\hat{Q}^{-3}\cdot(\hat{r}^2+\hat{Q})\cdot(4\hat{r}^2+\hat{Q})$.

Proof: By property P.2 and Proposition 4, the derivative of V_k along the closed-loop system dynamics can be upper bounded as follows:

$$\begin{aligned}\dot{V}_k &= \frac{1}{2}\sum_{i=1}^N\left(\dot{\mathbf{x}}_i^\mathsf{T}\dot{\mathbf{M}}_i\dot{\mathbf{x}}_i + 2\dot{\mathbf{x}}_i^\mathsf{T}\mathbf{M}_i\ddot{\mathbf{x}}_i + 2\dot{\hat{\mathbf{x}}}_i^\mathsf{T}\hat{\mathbf{M}}_i\ddot{\hat{\mathbf{x}}}_i\right)\\
&= \sum_{i=1}^N\dot{\mathbf{x}}_i^\mathsf{T}\mathrm{Sat}_i\left(-P_i\tilde{\mathbf{x}}_i - K_i\dot{\mathbf{x}}_i\right) + \sum_{i=1}^N\dot{\hat{\mathbf{x}}}_i^\mathsf{T}\mathrm{Sat}_i\left(P_i\tilde{\mathbf{x}}_i\right)\\
&\quad -\sum_{i=1}^N\dot{\hat{\mathbf{x}}}_i^\mathsf{T}\sum_{j\in\mathcal{N}_i}\nabla_i\hat{\psi}\left(\|\hat{\mathbf{x}}_{ij}^d\|\right) - \sum_{i=1}^N\hat{K}_i\|\dot{\hat{\mathbf{x}}}_i\|^2\\
&\leq \sum_{i=1}^N\left(\dot{\mathbf{x}}_i^\mathsf{T}\mathrm{Sat}_i\left(-P_i\tilde{\mathbf{x}}_i\right) + \dot{\hat{\mathbf{x}}}_i^\mathsf{T}\mathrm{Sat}_i\left(P_i\tilde{\mathbf{x}}_i\right)\right)\\
&\quad -\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\dot{\hat{\mathbf{x}}}_i^\mathsf{T}\nabla_i\hat{\psi}\left(\|\hat{\mathbf{x}}_{ij}^d\|\right) - \sum_{i=1}^N\hat{K}_i\|\dot{\hat{\mathbf{x}}}_i\|^2\\
&= -\sum_{i=1}^N\dot{\tilde{\mathbf{x}}}_i^\mathsf{T}\mathrm{Sat}_i\left(P_i\tilde{\mathbf{x}}_i\right) - \sum_{i=1}^N\hat{K}_i\|\dot{\hat{\mathbf{x}}}_i\|^2\\
&\quad -\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\dot{\hat{\mathbf{x}}}_i^\mathsf{T}\nabla_i\hat{\psi}\left(\|\hat{\mathbf{x}}_{ij}^d\|\right).\end{aligned}\tag{47}$$

Assumption 3 and the definitions of $\hat{\psi}(\|\hat{\mathbf{x}}_{ij}\|)$ and $\phi_i(\tilde{\mathbf{x}}_i)$ lead to

$$\begin{aligned}\dot{V}_p &= \frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i(0)}\left(\dot{\hat{\mathbf{x}}}_i^\mathsf{T}\nabla_i\hat{\psi}\left(\|\hat{\mathbf{x}}_{ij}\|\right) + \dot{\hat{\mathbf{x}}}_j^\mathsf{T}\nabla_j\hat{\psi}\left(\|\hat{\mathbf{x}}_{ij}\|\right)\right)\\
&\quad + \sum_{i=1}^N\dot{\tilde{\mathbf{x}}}_i^\mathsf{T}\mathrm{Sat}_i\left(P_i\tilde{\mathbf{x}}_i\right)\\
&= \sum_{i=1}^N\sum_{j\in\mathcal{N}_i(0)}\dot{\hat{\mathbf{x}}}_i^\mathsf{T}\nabla_i\hat{\psi}\left(\|\hat{\mathbf{x}}_{ij}\|\right) + \sum_{i=1}^N\dot{\tilde{\mathbf{x}}}_i^\mathsf{T}\mathrm{Sat}_i\left(P_i\tilde{\mathbf{x}}_i\right).\end{aligned}\tag{48}$$

Constant delay upper bounds $\overline{T}_{ji}$ indicate that

$$\dot{V}_d = \sum_{i=1}^N\sum_{j\in\mathcal{N}_i(0)}\overline{T}_{ji}\cdot\left(\|\dot{\hat{\mathbf{x}}}_j(t)\|^2 - \|\dot{\hat{\mathbf{x}}}_j\left(t-\overline{T}_{ji}\right)\|^2\right). \tag{49}$$

Because $\mathcal{N}_i(t) = \mathcal{N}_i(0)\ \forall i = 1, \ldots, N$ at time $t = 0$, and $V(\tau) \leq V(0)\ \forall\tau \in [0,t]$ implies $\mathcal{N}_i(t) = \mathcal{N}_i(0)$ and $\|\tilde{\mathbf{x}}_i(t)\| \leq (\epsilon/3)\ \forall t \geq 0$ and $\forall i = 1, \ldots, N$ by Lemma 2, set invariance [53] can again be leveraged to prove connectivity maintenance by induction on time [25]. Let $V(\tau) \leq V(0)\ \forall\tau \in [0,t)$ and thus $\mathcal{N}_i = \mathcal{N}_i(0)\ \forall\tau \in [0,t)$ in (47). It then suffices to prove $V(t) \leq V(0)$ as follows.

The definition of $\mathcal{N}_i$ implies that $|\hat{\mathbf{x}}_{ij}|_T < \hat{r}$ and, thus, that $\|\hat{\mathbf{x}}_{ij}\| < \hat{r}$ and $\|\hat{\mathbf{x}}_{ij}^d\| < \hat{r}$. Then, similar to (20)

$$\dot{\hat{\mathbf{x}}}_i^\mathsf{T}\nabla_i\hat{\psi}\left(\|\hat{\mathbf{x}}_{ij}\|\right) - \dot{\hat{\mathbf{x}}}_i^\mathsf{T}\nabla_i\hat{\psi}\left(\|\hat{\mathbf{x}}_{ij}^d\|\right) \leq \hat{\upsilon}\cdot\|\dot{\hat{\mathbf{x}}}_i\|\cdot\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_{jd}\| \tag{50}$$

holds $\forall j \in \mathcal{N}_i(0)$. Using (50) in the sum of (47)–(49) yields

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^N\sum_{j\in\mathcal{N}_i(0)}\left[\dot{\hat{\mathbf{x}}}_i^\mathsf{T}\nabla_i\hat{\psi}\left(\|\hat{\mathbf{x}}_{ij}\|\right) - \dot{\hat{\mathbf{x}}}_i^\mathsf{T}\nabla_i\hat{\psi}\left(\left\|\hat{\mathbf{x}}_{ij}^d\right\|\right)\right]\\
&\quad -\sum_{i=1}^N\hat{K}_i\cdot\|\dot{\hat{\mathbf{x}}}_i\|^2 + \sum_{i=1}^N\sum_{j\in\mathcal{N}_i(0)}\hat{\omega}\overline{T}_{ji}\cdot\|\dot{\hat{\mathbf{x}}}_j(t)\|^2\\
&\leq \sum_{i=1}^N\sum_{j\in\mathcal{N}_i(0)}\hat{\upsilon}\cdot\|\dot{\hat{\mathbf{x}}}_i\|\cdot\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_{jd}\|\\
&\quad -\sum_{i=1}^N\left(\hat{K}_i - \sum_{j\in\mathcal{N}_i(0)}\hat{\omega}\overline{T}_{ij}\right)\cdot\|\dot{\hat{\mathbf{x}}}_i\|^2.\end{aligned}\tag{51}$$

Applying the Cauchy–Schwarz inequality as in (22) leads to

$$\begin{aligned}&\int_0^t\|\dot{\hat{\mathbf{x}}}_i(\theta)\|\cdot\|\hat{\mathbf{x}}_j(\theta) - \hat{\mathbf{x}}_{jd}(\theta)\|d\theta\\
&\quad\leq \frac{\overline{T}_{ji}}{2}\cdot\left(\int_0^t\|\dot{\hat{\mathbf{x}}}_i(\theta)\|^2d\theta + \int_0^t\|\dot{\hat{\mathbf{x}}}_j(\theta)\|^2d\theta\right).\end{aligned}\tag{52}$$

Then, the time integration of (51) from 0 to t together with (52) lead to the following upper bound on $V(t)$:

$$\begin{aligned} V(t) &\leq \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i(0)} \frac{\hat{\upsilon}\overline{T}_{ji}}{2}\cdot\int_0^t\left(\|\dot{\hat{\mathbf{x}}}_i(\theta)\|^2+\|\dot{\hat{\mathbf{x}}}_j(\theta)\|^2\right)d\theta \\ &\quad -\sum_{i=1}^{N}\left(\hat{K}_i-\sum_{j\in\mathcal{N}_i(0)}\hat{\omega}\overline{T}_{ij}\right)\cdot\int_0^t\|\dot{\hat{\mathbf{x}}}_i(\theta)\|^2d\theta+V(0) \\ &= V(0)-\sum_{i=1}^{N}\tilde{K}_i\cdot\int_0^t\|\dot{\hat{\mathbf{x}}}_i(\theta)\|^2d\theta. \end{aligned} \tag{53}$$

Because $\tilde{K}_i$ are positive by (46), (53) guarantees that $V(t) \leq V(0)$ and, by Lemma 2, that all initial links are preserved.

Synchronization can be concluded by noting that (53) also yields that $\dot{\hat{\mathbf{x}}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, which implies that $\dot{\hat{\mathbf{x}}}_i \to \mathbf{0}$. After obtaining the time derivative of $\ddot{\hat{\mathbf{x}}}_i$ in (24), Barbalat's lemma yields $\ddot{\hat{\mathbf{x}}}_i \to \mathbf{0}$. The bounded second derivative of $\ddot{\hat{\mathbf{x}}}_i$ leads to $\dddot{\hat{\mathbf{x}}}_i \to \mathbf{0}$ and further to $\dot{\hat{\mathbf{x}}}_i \to \dot{\mathbf{x}}_i \to \mathbf{0}$ and $\hat{\mathbf{x}}_{ij} \to \mathbf{0}$. The derivative of (1) gives $\ddot{\mathbf{x}}_i \to \mathbf{0}$ and thus $\mathrm{Sat}_i\big(P_i(\hat{\mathbf{x}}_i - \mathbf{x}_i)\big) \to \mathbf{0}$. Together, all the above inferences lead to $\tilde{\mathbf{x}}_i \to \mathbf{0}$ and $\hat{\mathbf{x}}_{ij} \to \mathbf{0}$, that is, to $\mathbf{x}_i \to \mathbf{x}_j$. ■

Remark 10: The first stepping-stone in the proposed indirect coupling framework is to connect the agents only to their proxies and through saturated P+d control. Single robot regulation control [50] provides the impetus for employing saturated P+d terms for agent control, by showing that the control exploits actuator capabilities better than other saturating controllers, potentially speeding up convergence. The virtual proxies decouple the actuator saturation and time-varying delays in agent-proxy and interproxy couplings, respectively, thereby permitting the control design to tackle them separately. The second stepping-stone of the framework is to convert the impact of limited actuation on connectivity maintenance to a constraint on the time-delay interproxy couplings. Then, a single Lyapunov candidate (37) can verify all constraints after synchronizing the upper bounds of their corresponding potential functions.

Remark 11: Existing work has proposed controllers for connectivity-preserving consensus of first-order [5] and second-order MASs [25]. Connectivity-preserving consensus of first-order networks with bounded actuation has been studied in [12]. To the authors' best knowledge, connectivity-preserving consensus of EL networks with time-varying delays and bounded actuation has remained an open question. The key challenge in this question is the saturated actuation. Given unlimited actuation, Section III-A has shown that proper modulation of the inter-agent couplings and sufficient local damping injection preserve the local connectivity of time-delay EL networks during synchronization. Actuator saturation, however, limits the coupling stiffness and the injected damping. Thereby, it prevents existing controllers from guaranteeing connectivity and, implicitly, synchronization.

Remark 12: This paper focuses on the connectivity-preserving synchronization of time-delay EL networks with undirected communications.

1) Given unbounded actuation (Section III-A), gradient-based controls couple initially adjacent agents symmetrically. In particular, the gradient term $-\nabla_i\psi(\|\mathbf{x}_{ij}^d\|)$ in the control $\mathbf{u}_i$ designed in (3) connects agent i to agent j. Correspondingly, the gradient term $-\nabla_j\psi(\|\mathbf{x}_{ji}^d\|)$ in the control $\mathbf{u}_j$ connects agent j to agent i because $i \in \mathcal{N}_j$ iff $j \in \mathcal{N}_i$ for undirected communications.
2) Given bounded actuation (Section III-B), saturated proportional controls interconnect each agent with its proxy while gradient-based controls couple proxies of initially adjacent agents symmetrically. In particular, the saturated proportional control $-P_i\tilde{\mathbf{x}}_i$ in (27) and $\mathrm{Sat}_i(P_i\tilde{\mathbf{x}}_i)$ in (24) interconnects agent i with its proxy. Further, the gradient terms $-\nabla_i\hat{\psi}(\|\hat{\mathbf{x}}_{ij}^d\|)$ and $-\nabla_j\hat{\psi}(\|\hat{\mathbf{x}}_{ji}^d\|)$ mutually couple the proxies of initially adjacent agents i and j according to (24). In this context, adjacent agents are interconnected/coupled indirectly, through their proxies.

Thus, this paper develops gradient-based controls that symmetrically couple initially adjacent agents, directly or indirectly, to preserve their undirected communication links.

Remark 13: The strategies in this paper do not extend to EL networks with directed communications directly. The asymmetry of agent couplings in directed networks poses the key obstacle. For example, if agent j receives information from agent i but not vice versa, agent j must remain sufficiently close to agent i to preserve the unidirectional communication link (i, j) while agent i does not know the state of agent j. Hence, agent i affects the motion of agent j but agent j cannot affect the motion of agent i. This is unlike the control strategies in this paper, which employ mutual and symmetric couplings, as discussed in Remark 12.

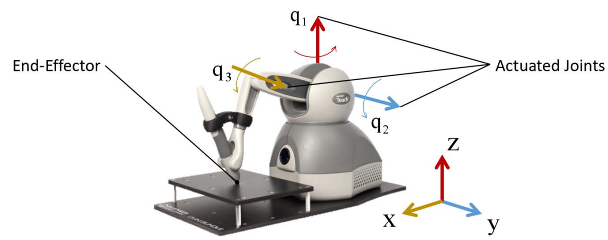


Fig. 1. A Geomagic Touch haptic robot.

IV. EXPERIMENTS

This section compares the controllers, proposed in (3) for time-delay EL networks with full actuation, and in (24) plus (27) for time-delay EL networks with limited actuation, to conventional P+d control [4] through experiments with $N = 3$ Geomagic Touch haptic robots. Each robot has three actuated joints as shown in Fig. 1 and thus, can drive its end-effector in 3-D task space. Each robot has an inner-loop controller for gravity compensation in joint space, and an outer-loop controller for synchronization in task space. The video of the experiment is available at https://youtu.be/P5sY_8l0ihw.

The robots are initially at rest at $\mathbf{x}_1(0) = (-0.15, -0.1, -0.1)^\mathsf{T}$ m, $\mathbf{x}_2(0) = (-0.15, 0.15, 0.12)^\mathsf{T}$ m, and $\mathbf{x}_3(0) = (-0.2, 0, -0.05)^\mathsf{T}$ m. Their communication radius is $r = 0.25$ m. Hence, agent 1 and agent 3,

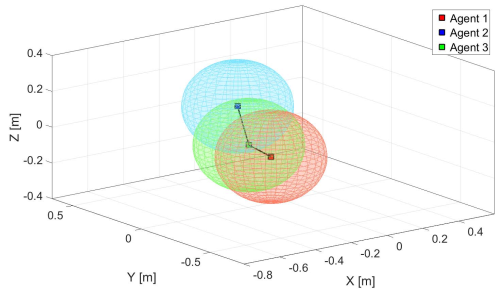
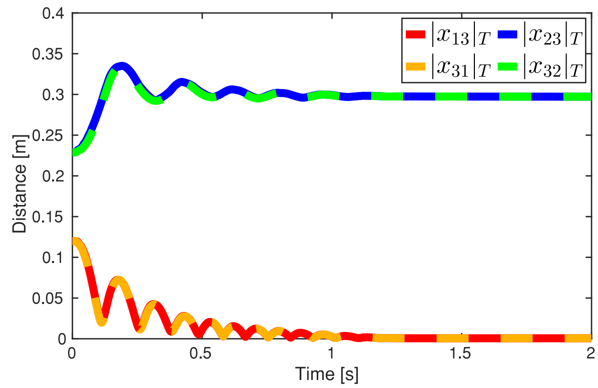
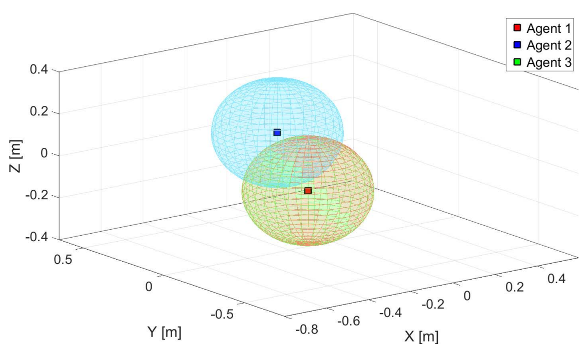


Fig. 2. Initial positions $\mathbf{x}_i(0)$ of the end-effectors of three robots $i = 1, 2, 3$. Their communication zones, each with radius r, are depicted as spheres centered around the initial positions. Black solid lines connect initially adjacent agents.



(a)



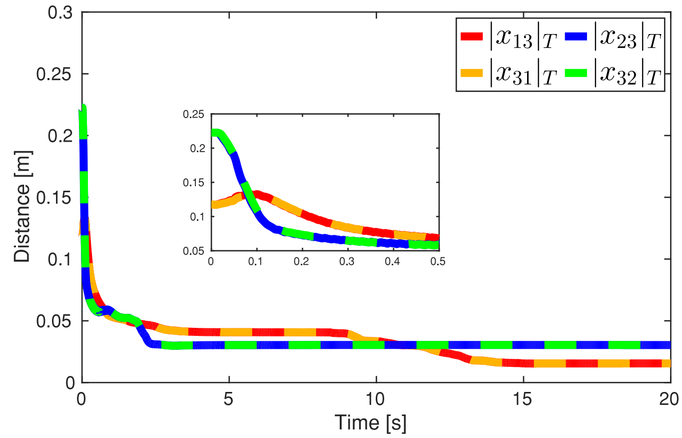
(b)

Fig. 3. Experimental interagent distances and steady-state positions of the end-effectors of three robots with communication distances $r = 0.25$ m under conventional P+d synchronization control [4]. The P+d controller cannot preserve the initial link (2, 3) and thus fails to synchronize the three end-effectors.

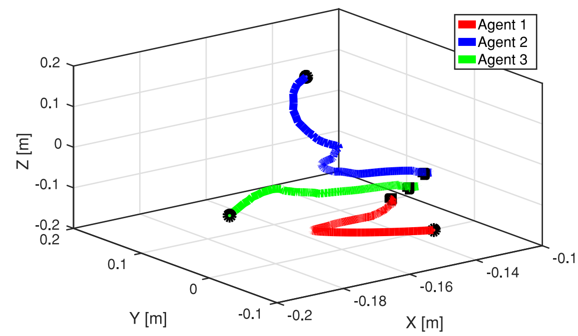
and agent 2 and agent 3 are initially adjacent, that is, $\mathcal{E}(0) = \{(1, 3), (2, 3)\}$. Fig. 2 depicts the initial positions of the end-effectors of three robots and the connectivity of the EL network. Time-varying communication delays $T_{ij}(t)$ are up to $\overline{T}_{ij} = 5$ ms.

A. Proportional Plus Damping Control [4]

Let the parameters giving the synchronizing P gains be $p_i = 1$, $\omega_{13} = \omega_{31} = 200$, and $\omega_{23} = \omega_{32} = 0.5$. If the agent communication distances are unlimited, conventional P+d control with damping gains $d_1 = 8$, $d_2 = 5$, and $d_3 = 3$ stably synchronizes the EL network. However, it cannot synchronize the three robots with the given communication radius $r = 0.25$ m (see Fig. 3). Because agent 3 has a much stiffer coupling to agent 1 than to agent 2, it moves away from agent 2 so quickly that the distance between them increases to about 0.3 m before agent 2 can track it. The controller cannot maintain the initial link (2, 3). As a result, the network looses connectivity. Agent 2 becomes isolated and the controller cannot synchronize it with the other two agents, as seen in Fig. 3. This experiment verifies that the connectivity of EL networks whose agents have limited communication distance must be guaranteed during synchronization.



(a)



(b)

Fig. 4. Experimental interagent distances and end-effectors paths of three robots with communication distances $r = 0.25$ m under the proposed gradient plus damping control (3). The controller preserves connectivity but loosely synchronizes the robots, with steady-state position errors smaller than 0.05 m.

B. Proposed Gradient Plus Damping Control in (3)

After setting $Q = 0.0047$ to satisfy condition (9), $P = 0.2$ is tuned heuristically to couple the initially adjacent agents. In the presence of friction in the joints and unreliable velocity measurements, the local damping gains which practically stabilize the robots are $K_1 = K_2 = 3$ and $K_3 = 6$. Fig. 4(a) verifies that the proposed gradient plus damping control (3) restricts the distances between agents 1 and 3, and between agents 2 and 3, below r. Hence, it preserves all initial links, (1, 3) and (2, 3), during synchronization. Its

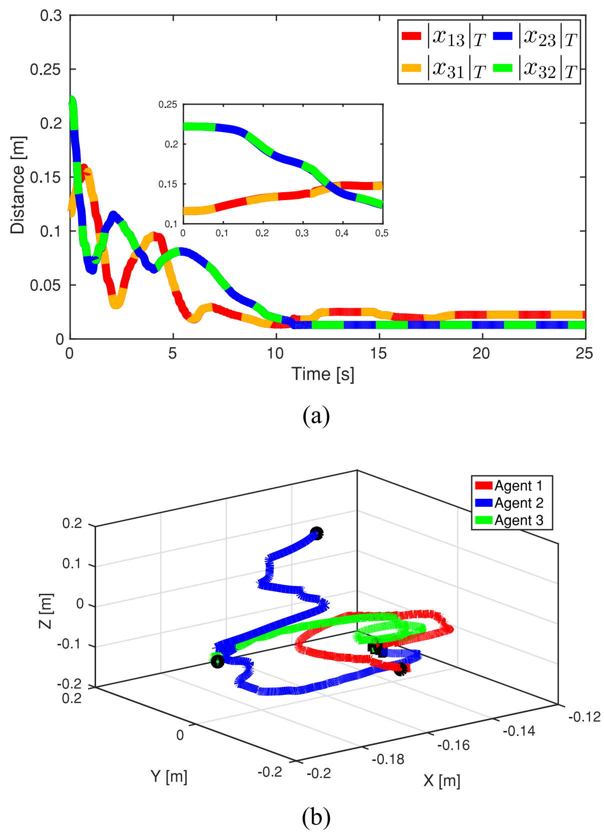


Fig. 5. Experimental interagent distances and end-effector paths of three robots with communication distances $r = 0.25$ m under the proposed indirect coupling strategy (24) and (27). The strategy preserves connectivity and practically better synchronizes the robots, with steady-state position errors smaller than 0.02 m.

gradient terms (4) make the interagent couplings more compliant when the interagent distances decrease. The parameters P and Q can reduce overshoots by weakening initially stiff couplings (see Fig. 4(a)). However, they make interagent couplings over-compliant when the interagent distances become small. Compliant interagent couplings demand relatively large position errors to balance gravity compensation errors and intrinsic friction in the robot joints. Therefore, the robots can only be driven closely to the others, with steady-state position errors smaller than 0.05 m, as depicted in Fig. 4.

C. Proposed Indirect Coupling Control in (24) and (27)

The control force in each direction is bounded by $\bar{u}_i^k = 0.5$ N for all robots. Each robot i is tightly connected to its proxy by $P_i = 500$ and $K_i = 5$. Given $\hat{r} = 0.24$ m, the selection $\hat{Q} = 0.0013$ guarantees condition (39). Then, $\hat{P} = 0.2$ and $\hat{K}_i = 24$ practically stabilize the system. All actuators of all robots saturate for a certain time during the first 10 s of the synchronization. The actuator force time histories are omitted here due to limited space, but are presented in the video of the experiments at https://youtu.be/P5sY_8l0ihw. Although the saturation of the actuators makes the interagent distances fluctuate during the first 10 s, the indirect coupling strategy maintains the distances between agents 1 and 3 [the red solid line and the orange dashed line in Fig. 5(a)], and between agents 2 and 3 [the blue solid line and the green dashed line in Fig. 5(a)], strictly smaller than their communication distance r and thus, preserves the connectivity of the network. Compared to Fig. 4(b), it synchronizes the robot end-effectors better, with steady-state errors reduced to about 0.02 m in Fig. 5(b). The parameters $\hat{P}$ and $\hat{Q}$ can be selected to stiffen the dynamic interproxy couplings even for small interproxy distances, and the virtual damping $\hat{K}_i$ can be injected to address the delay-induced instability independent of unreliable robot velocity measurements.

V. Conclusion

This paper has investigated the impact of time-varying delays and actuator saturation on the connectivity-preserving synchronization of EL networks. For time-delay EL networks with unbounded actuation, this paper has proposed a gradient plus damping control that modulates the agent interconnections dynamically and injects damping locally to preserve connectivity during synchronization. For time-delay EL networks with bounded actuation, this paper has developed an indirect coupling strategy that decomposes the interagent couplings into agent-proxy and interproxy couplings, and overcomes actuator saturation in the former and time-varying delays in the latter. Experiments with three Geomagic Touch haptic robots have validated the proposed designs.

The connectivity of practical multirobot networks is also threatened by uncertain perturbations. Future work will investigate the robust connectivity maintenance for perturbed EL networks, especially for the human-in-the-loop swarms.

References

[1] W. Ren and R. W. Beard, “Consensus seeking in multiagent systems under dynamically changing interaction topologies,” *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655–661, May 2005.

[2] Z. Lin, M. Broucke, and B. Francis, “Local control strategies for groups of mobile autonomous agents,” *IEEE Trans. Autom. Control*, vol. 49, no. 4, pp. 622–629, Apr. 2004.

[3] W. Ren, “On consensus algorithms for double-integrator dynamics,” *IEEE Trans. Autom. Control*, vol. 53, no. 6, pp. 1503–1509, Jul. 2008.

[4] E. Nuño, I. Sarras, and L. Basañez, “Consensus in networks of non-identical Euler–Lagrange systems using P+d controllers,” *IEEE Trans. Robot.*, vol. 29, no. 6, pp. 1503–1508, Dec. 2013.

[5] M. M. Zavlanos, M. B. Egerstedt, and G. J. Pappas, “Graph-theoretic connectivity control of mobile robot networks,” *Proc. IEEE*, vol. 99, no. 9, pp. 1525–1540, Sep. 2011.

[6] M. Ji and M. Egerstedt, “Distributed coordination control of multiagent systems while preserving connectedness,” *IEEE Trans. Robot.*, vol. 23, no. 4, pp. 693–703, Aug. 2007.

[7] D. V. Dimarogonas and K. J. Kyriakopoulos, “On the rendezvous problem for multiple nonholonomic agents,” *IEEE Trans. Autom. Control*, vol. 52, no. 5, pp. 916–922, May 2007.

[8] D. V. Dimarogonas and K. J. Kyriakopoulos, “Connectedness preserving distributed swarm aggregation for multiple kinematic robots,” *IEEE Trans. Robot.*, vol. 24, no. 5, pp. 1213–1223, Oct. 2008.

[9] M. M. Zavlanos and G. J. Pappas, “Distributed connectivity control of mobile networks,” *IEEE Trans. Robot.*, vol. 24, no. 6, pp. 1416–1428, Dec. 2008.

[10] A. Ajorlou, A. Momeni, and A. G. Aghdam, “A class of bounded distributed control strategies for connectivity preservation in multi-agent systems,” *IEEE Trans. Autom. Control*, vol. 55, no. 12, pp. 2828–2833, Dec. 2010.

[11] D. V. Dimarogonas and K. H. Johansson, “Decentralized connectivity maintenance in mobile networks with bounded inputs,” in *Proc. IEEE Int. Conf. Robot. Autom.*, May 2008, pp. 1507–1512.

[12] A. Ajorlou and A. G. Aghdam, "Connectivity preservation in nonholonomic multi-agent systems: A bounded distributed control strategy," *IEEE Trans. Autom. Control*, vol. 58, no. 9, pp. 2366–2371, Sep. 2013.

[13] Z. Kan, L. Navaravong, J. M. Shea, E. L. Pasiliao, and W. E. Dixon, "Graph matching-based formation reconfiguration of networked agents with connectivity maintenance," *IEEE Trans. Control Netw. Syst.*, vol. 2, no. 1, pp. 24–35, Mar. 2015.

[14] J.-G. Dong, "Finite-time connectivity preservation rendezvous with disturbance rejection," *Automatica*, vol. 71, pp. 57–61, Sep. 2016.

[15] Y. Cao, W. Ren, D. W. Casbeer, and C. Schumacher, "Finite-time connectivity-preserving consensus of networked nonlinear agents with unknown Lipschitz terms," *IEEE Trans. Autom. Control*, vol. 61, no. 6, pp. 1700–1705, Jun. 2016.

[16] I. Saboori, H. Nayyeri, and K. Khorasani, "A distributed control strategy for connectivity preservation of multi-agent systems subject to actuator saturation," in *Proc. Amer. Control Conf.*, Jun. 2013, pp. 4044–4049.

[17] Z. Kan, A. P. Dani, J. M. Shea, and W. E. Dixon, "Network connectivity preserving formation stabilization and obstacle avoidance via a decentralized controller," *IEEE Trans. Autom. Control*, vol. 57, no. 7, pp. 1827–1832, Jul. 2012.

[18] S. Zhao, D. V. Dimarogonas, Z. Sun, and D. Bauso, "A general approach to coordination control of mobile agents with motion constraints," *IEEE Trans. Autom. Control*, vol. 63, no. 5, pp. 1509–1516, May 2018.

[19] H. A. Poonawala and M. W. Spong, "Preserving strong connectivity in directed proximity graphs," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4392–4404, Sep. 2017.

[20] G. A. Hollinger and S. Singh, "Multirobot coordination with periodic connectivity: Theory and experiments," *IEEE Trans. Robot.*, vol. 28, no. 4, pp. 967–973, Aug. 2012.

[21] J. Banfi, A. Q. Li, N. Basilico, I. Rekleitis, and F. Amigoni, "Asynchronous multirobot exploration under recurrent connectivity constraints," in *Proc. IEEE Int. Conf. Robot. Autom. (ICRA)*, May 2016, pp. 5491–5498.

[22] Y. Kantaros and M. M. Zavlanos, "Distributed intermittent connectivity control of mobile robot networks," *IEEE Trans. Autom. Control*, vol. 62, no. 7, pp. 3109–3121, Jul. 2017.

[23] D. Boskos and D. V. Dimarogonas, "Robustness and invariance of connectivity maintenance control for multiagent systems," *SIAM J. Control Optim.*, vol. 55, no. 3, pp. 1887–1914, 2017.

[24] A. Gasparri, L. Sabattini, and G. Ulivi, "Bounded control law for global connectivity maintenance in cooperative multirobot systems," *IEEE Trans. Robot.*, vol. 33, no. 3, pp. 700–717, Jun. 2017.

[25] H. Su, X. Wang, and G. Chen, "Rendezvous of multiple mobile agents with preserved network connectivity," *Syst. Control Lett.*, vol. 59, no. 5, pp. 313–322, 2010.

[26] H. Su, G. Chen, X. Wang, and Z. Lin, "Adaptive second-order consensus of networked mobile agents with nonlinear dynamics," *Automatica*, vol. 47, no. 2, pp. 368–375, 2011.

[27] Y. Dong and J. Huang, "A leader-following rendezvous problem of double integrator multi-agent systems," *Automatica*, vol. 49, no. 5, pp. 1386–1391, 2013.

[28] Y. Dong and J. Huang, "Leader-following connectivity preservation rendezvous of multiple double integrator systems based on position measurement only," *IEEE Trans. Autom. Control*, vol. 59, no. 9, pp. 2598–2603, Sep. 2014.

[29] Y. Su, "Leader-following rendezvous with connectivity preservation and disturbance rejection via internal model approach," *Automatica*, vol. 57, pp. 203–212, Jul. 2015.

[30] L. Wang, X. Wang, and X. Hu, "Connectivity maintenance and distributed tracking for double-integrator agents with bounded potential functions," *Int. J. Robust Nonlin. Control*, vol. 25, no. 4, pp. 542–558, 2015.

[31] Z. Feng, C. Sun, and G. Hu, "Robust connectivity preserving rendezvous of multirobot systems under unknown dynamics and disturbances," *IEEE Trans. Control Netw. Syst.*, vol. 4, no. 4, pp. 725–735, Dec. 2017.

[32] X. Ai, S. Song, and K. You, "Second-order consensus of multi-agent systems under limited interaction ranges," *Automatica*, vol. 68, pp. 329–333, Jun. 2016.

[33] Z. Meng, Z. Lin, and W. Ren, "Leader–follower swarm tracking for networked Lagrange systems," *Syst. Control Lett.*, vol. 61, no. 1, pp. 117–126, 2012.

[34] Y. Dong and J. Huang, "Leader-following consensus with connectivity preservation of uncertain Euler–Lagrange multi-agent systems," *Int. J. Robust Nonlin. Control*, vol. 27, no. 18, pp. 4772–4787, 2017.

[35] Z. Chen and H.-T. Zhang, "Analysis of joint connectivity condition for multiagents with boundary constraints," *IEEE Trans. Cybern.*, vol. 43, no. 2, pp. 437–444, Apr. 2013.

[36] Y. Dong and J. Huang, "Flocking with connectivity preservation of multiple double integrator systems subject to external disturbances by a distributed control law," *Automatica*, vol. 55, pp. 197–203, May 2015.

[37] C. Sun, G. Hu, L. Xie, and M. Egerstedt, "Robust finite-time connectivity preserving coordination of second-order multi-agent systems," *Automatica*, vol. 89, pp. 21–27, Mar. 2018.

[38] S. J. Yoo and B. S. Park, "Connectivity-preserving approach for distributed adaptive synchronized tracking of networked uncertain nonholonomic mobile robots," *IEEE Trans. Cybern.*, vol. 48, no. 9, pp. 2598–2608, Sep. 2018.

[39] L. Sabattini, N. Chopra, and C. Secchi, "Decentralized connectivity maintenance for cooperative control of mobile robotic systems," *Int. J. Robot. Res.*, vol. 32, no. 12, pp. 1411–1423, 2013.

[40] P. R. Giordano, A. Franchi, C. Secchi, and H. H. Bülthoff, "A passivity-based decentralized strategy for generalized connectivity maintenance," *Int. J. Robot. Res.*, vol. 32, no. 3, pp. 299–323, 2013.

[41] L. Sabattini, C. Secchi, N. Chopra, and A. Gasparri, "Distributed control of multirobot systems with global connectivity maintenance," *IEEE Trans. Robot.*, vol. 29, no. 5, pp. 1326–1332, Oct. 2013.

[42] L. Sabattini, C. Secchi, and N. Chopra, "Decentralized estimation and control for preserving the strong connectivity of directed graphs," *IEEE Trans. Cybern.*, vol. 45, no. 10, pp. 2273–2286, Oct. 2015.

[43] Z. Chen, M.-C. Fan, and H.-T. Zhang, "How much control is enough for network connectivity preservation and collision avoidance?" *IEEE Trans. Cybern.*, vol. 45, no. 8, pp. 1647–1656, Aug. 2015.

[44] R. K. Williams and G. S. Sukhatme, "Constrained interaction and coordination in proximity-limited multiagent systems," *IEEE Trans. Robot.*, vol. 29, no. 4, pp. 930–944, Aug. 2013.

[45] R. K. Williams, A. Gasparri, G. Ulivi, and G. S. Sukhatme, "Generalized topology control for nonholonomic teams with discontinuous interactions," *IEEE Trans. Robot.*, vol. 33, no. 4, pp. 994–1001, Aug. 2017.

[46] H. Fang, Y. Wei, J. Chen, and B. Xin, "Flocking of second-order multiagent systems with connectivity preservation based on algebraic connectivity estimation," *IEEE Trans. Cybern.*, vol. 47, no. 4, pp. 1067–1077, Apr. 2017.

[47] A. Abdessameud, A. Tayebi, and I. G. Polushin, "Attitude synchronization of multiple rigid bodies with communication delays," *IEEE Trans. Autom. Control*, vol. 57, no. 9, pp. 2405–2411, Sep. 2012.

[48] E. Nuño, R. Ortega, L. Basañez, and D. Hill, "Synchronization of networks of nonidentical Euler–Lagrange systems with uncertain parameters and communication delays," *IEEE Trans. Autom. Control*, vol. 56, no. 4, pp. 935–941, Apr. 2011.

[49] E. Nuño and R. Ortega, "Achieving consensus of Euler–Lagrange agents with interconnecting delays and without velocity measurements via passivity-based control," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 1, pp. 222–232, Jan. 2018.

[50] A. Zavala-Río and V. Santibáñez, "A natural saturating extension of the PD-with-desired-gravity-compensation control law for robot manipulators with bounded inputs," *IEEE Trans. Robot.*, vol. 23, no. 2, pp. 386–391, Apr. 2007.

[51] R. Kelly, V. S. Davila, and J. A. L. Perez, *Control of Robot Manipulators in Joint Space*. London, U.K.: Springer, 2006.

[52] Y. Yang, D. Constantinescu, and Y. Shi, "Connectivity-preserving consensus of multi-agent systems with bounded actuation," *arXiv preprint arXiv:1803.09309*, 2018.

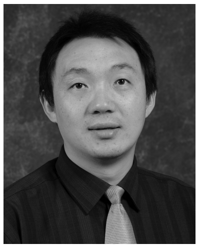
[53] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747–1767, 1999.

[54] F. Morabito, A. R. Teel, and L. Zaccarian, "Nonlinear antiwindup applied to Euler–Lagrange systems," *IEEE Trans. Robot. Autom.*, vol. 20, no. 3, pp. 526–537, Jun. 2004.

[55] A. Zavala-Río and V. Santibáñez, "Simple extensions of the PD-with-gravity-compensation control law for robot manipulators with bounded inputs," *IEEE Trans. Control Syst. Technol.*, vol. 14, no. 5, pp. 958–965, Sep. 2006.

Yuan Yang received the B.Eng. degree in mechanical design, manufacturing and automation from the Harbin Institute of Technology, Harbin, China, in 2015 and the M.A.Sc. degree in mechanical engineering from the University of Victoria, Victoria, BC, Canada, in 2017, where he is currently pursuing the Ph.D. degree in mechanical engineering.

His current research interests include distributed control of networked dynamical systems, human–robot collaboration, and safety for physical human–robot interaction.



Yang Shi (SM'09–F'17) received the Ph.D. degree in electrical and computer engineering from the University of Alberta, Edmonton, AB, Canada, in 2005.

From 2005 to 2009, he was a Faculty Member with the Department of Mechanical Engineering, University of Saskatchewan, Saskatoon, SK, Canada. He is currently a Professor with the Department of Mechanical Engineering, University of Victoria, Victoria, BC, Canada. His current research interests include networked and distributed systems, model predictive control, industrial cyber-physical systems, mechatronics and robotics, and energy systems.

Prof. Shi was a recipient of the University of Saskachewan Student Union Teaching Excellence Award in 2007, the Faculty of Engineering Teaching Excellence Award from the University of Victoria in 2012, the JSPS Invitation Fellowship (short-term) in 2013, and the 2015 Craigdarroch Silver Medal for Excellence in Research from the University of Victoria. He currently serves as the Co-Editor-in-Chief for the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS and an Associate Editor for *Automatica*, the IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, the IEEE/ASME TRANSACTIONS ON MECHATRONICS, and the IEEE TRANSACTIONS ON CYBERNETICS. He is the Chair of IEEE Industrial Electronics Society Technical Committee on Industrial Cyber-Physical Systems. He is a fellow of the Canadian Society for Mechanical Engineering, American Society of Mechanical Engineers, and Engineering Institute of Canada, and a Registered Professional Engineer in the Province of British Columbia, Canada.

Daniela Constantinescu (M'99) received the Ph.D. degree in electrical and computer engineering from the University of British Columbia, Vancouver, BC, Canada, in 2005.

She has been with the University of Victoria, Victoria, BC, Canada, since 2005, where she is currently an Associate Professor with the Department of Mechanical Engineering. Her current research interests include teleoperation and haptic systems, distributed control for network robotic systems, human–robot interaction, and robotics and dynamics.