Distributed Connectivity-Preserving Coordination of Multi-agent Systems with Bounded Velocities

Chapter · January 2020

DOI: 10.1007/978-3-030-25629-6_108

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Abstract. This paper presents a distributed control strategy for the coordination with local connectivity maintenance of single-integrator multi-agent systems with bounded velocities. The proposed strategy regards actuator saturation as dynamic scaling of the control. It preserves connectivity by applying gradient-based controls that can monotonically decrease suitably designed local potentials even if saturated. As a result, the controller design is not constrained by the velocity bounds, and can fully exploit the limited velocities for connectivity-preserving coordination. Numerical simulations verify the performance of the proposed controller.

Keywords: Multi-agent systems · Connectivity-preserving coordination · Bounded actuation

1 Introduction

Distributed coordination control of multi-agent systems ~ (MAS), whose objective is to drive the multiple agents to a common configuration using only local information exchanges, has been extensively studied [1]. While initial coordination techniques have assumed MAS connectivity [2], more recent strategies have used gradient-based controls to maintain it locally [3–6].

One limitation of gradient-based controls is that they may exceed the actuator bounds or grow unbounded unless the local potentials, which yield the controls, are carefully designed. Local potentials to overcome this limitation have been used in [7–9]. Yet, potentials guaranteeing bounded gradient-based controls are not enough to preserve MAS connectivity. Actuator saturation may still distort the control input and lead to loss of connectivity. The dynamic compensation strategy in [9] has preserved system connectivity by limiting the local controls within actuation bounds, but has increased the order of the system and is conservative when the control signals approach their bounds.

This paper introduces a distributed coordination controller for single-integrator MAS that both maintain the local connectivity and better exploit the bounded velocities. It proposes smooth and bounded local potentials which yield gradient-based controls that behave as Proportional ~ (P) controllers with distance-dependent gains. Innovatively, the new local control laws limit the effect of actuator saturation to a

T. Ahram et al. (Eds.): IHIET 2019, AISC 1018, pp. 695–699, 2020.
https://doi.org/10.1007/978-3-030-25629-6_108

dynamic scaling of the P controls by time-varying positive gains. Hence, the bounded gradient-based controls still implement a distributed P law with time-varying, positive and lower-bounded P gains that decrease the local potentials monotonically while maintaining the sensing links of the MAS. Two sets of numerical simulations verify the performance of the proposed controllers for single-integrator a MAS with bounded velocities.

2 Preliminaries

In a MAS, each agent i, with position $\mathbf{p}_i \in \Re^n$, has limited communication capability. Two agents i and j are adjacent/neighbors if and only if they can, and agree to, exchange information with each other. To be able to exchange information, they must be within their communication distance r. Note that if agent j is within the communication distance of i, $\|\mathbf{p}_{ij}\| = \|\mathbf{p}_j - \mathbf{p}_i\| \leq r$, then agent i is also within the communication distance of j, $\|\mathbf{p}_{ji}\| = \|\mathbf{p}_i - \mathbf{p}_j\| \leq r$. However, agents within communication distance of each other are neighbors only if they agree to exchange information with each other. Therefore, the set of neighbors of any agent need not change when coordination is achieved and all agents are within communication distance of all other agents.

This paper adopts the following definition of, and assumptions on, of the communication graph of a MAS.

Definition 1. [10] The communication graph of a MAS $G = \{v, E\}$ consists of a set of nodes $v = \{1, \ldots, N\}$, each associated with a different agent in the system, and a set of communication edges $E = \{(i,j) \in v \times v | i \in N_j\}$, each associated with a communication link in the system, where N_j is the set of neighbors of agent j.

Assumption 1. The communication graph is undirected, i.e., $(i,j) \in E$ iff $(j,i) \in E$.

Assumption 2. The undirected communication graph of the MAS is initially connected and each pair of initially adjacent agents (i,j) is strictly within their communication distance, i.e., $\|\mathbf{p}_{ij}(0)\| = \|\mathbf{p}_{ji}(0)\| \leq r - \varepsilon$ for some $\varepsilon > 0$.

The paper further adopts the definitions in [10] of the weighted adjacency matrix $\mathbf{A}$, weighted Laplacian matrix $\mathbf{L}$, and incidence matrix $\mathbf{D}$ associated with a graph G. Hence, $\mathbf{A}$ is symmetric, and $\mathbf{L}$ and $\mathbf{D}$ are related via $\mathbf{L} = \mathbf{DWD}^{\mathrm{T}}$[10], where $\mathbf{W}$ is an $M \times M$ diagonal matrix with the diagonal entries equal to the weights of each edge and M the number of edges of graph G.

3 Main Result

Consider an MAS of N single-integrator agents with dynamics:

$$\dot{\mathbf{p}}_i = \mathbf{u}_i \tag{1}$$

where: $i = 1, \ldots, N$ indexes the agents in the system; $\mathbf{p}_i = (p_{i1} \ldots p_{in})^{\mathrm{T}} \in \Re^n$ is the position vector of agent i; and $\mathbf{u}_i = (u_{i1}, \ldots, u_{in})^{\mathrm{T}} \in \Re^n$ is the actual control of agent i.

For each pair of initially adjacent agents $(i,j) \in E$, consider the potential function:

$$\varphi(\|\mathbf{p}_{ij}\|) = \frac{\|\mathbf{p}_{ij}\|^2}{r^2 - \|\mathbf{p}_{ij}\|^2 + \frac{r^2}{Q}} \tag{2}$$

where Q is a positive constant selected to obey $Q > \frac{N(N-1)(r-\varepsilon)^2}{2\left(r^2-(r-\varepsilon)^2+\frac{r^2}{Q}\right)}$. By Assumption 2, all $\varphi(\|\mathbf{p}_{ij}\|)$ are defined on $[0, r]$ at $t = 0$.

The following gradient-based control law serves to coordinate the MAS and preserve its connectivity in this paper:

$$\tilde{\mathbf{u}}_i = -\Sigma_{j \in N_i} \nabla_i \varphi(\|\mathbf{p}_{ij}\|) \tag{3}$$

where N_i is the set of neighbours of agent i at time $t = 0$, and $\nabla_i \varphi(\|\mathbf{p}_{ij}\|)$ is the gradient of $\varphi(\|\mathbf{p}_{ij}\|)$ with respect to $\mathbf{p}_i$. The actual control $\mathbf{u}_i$ on agent i is equal to the designed control $\tilde{\mathbf{u}}_i$ if $\tilde{\mathbf{u}}_i$ is within the agent's actuation bound; otherwise, the agent's actuator saturates and applies only part of $\tilde{\mathbf{u}}_i$:

$$\mathbf{u}_i = \mathrm{Sat}_i(\tilde{\mathbf{u}}_i) = (\mathrm{sat}_{i1}(\tilde{u}_{i1})\ldots\mathrm{sat}_{in}(\tilde{u}_{in}))^{\mathrm{T}} \tag{4}$$

The connectivity preservation and coordination of single-integrator MAS under the bounded gradient-based control law in (4) can be investigated by regarding actuator saturation as scaling of the designed control through time-varying positive gains:

$$\mathbf{u}_i = \mathrm{Sat}(\tilde{\mathbf{u}}_i) = \mathbf{S}_i(t)\tilde{\mathbf{u}}_i \tag{5}$$

where $\mathbf{S}_i = \mathrm{diag}\{s_{i1}(t), \ldots, s_{in}(t)\}, i = 1, \ldots, N$ are diagonal matrices with diagonal elements $s_{ik}(t) > 0$ for all $k = 1, \ldots, n$.

The Lyapunov function that facilitates the proof that the bounded control (5) maintains the initial communication links of the single-integrator MAS (1) is designed as:

$$V = \frac{1}{2}\Sigma_{i=1,\ldots,N}\Sigma_{j \in N_i}\varphi(\|\mathbf{p}_{ij}\|) \tag{6}$$

Algebraic manipulations show that the derivative of this function is non-positive and, thus, that is V monotonically decreasing, i.e., $V(t) \leq V(0)$, for all $t \geq 0$. The same function V, together with the Lasalle Invariance Principle, can be used to deduce coordination. Both proofs are omitted here due to page limits.

4 Simulation Results

This section compares the controller in (3) to that in [9] through simulations of a MAS with $N = 5$ single-integrator agents with: dimension $n = 1$; communication radius $r = 1$ m; actuation bounds 2 m/s, 1 m/s, 1 m/s, 2 m/s and 3 m/s; and initial position

1 m, 1.5 m, 2 m, 2.5 m and 3 m. For the controller in (6), condition (9) is guaranteed by $Q = 40$ after selecting $\varepsilon = 0.1$ m. For the controller in [9], the design parameters are $\alpha_i = 1$, $\varepsilon = 0.1$ and $\beta_i = 11$.

Figures 1 and 2 illustrate that the MAS is coordinated in about 0.7 s by the controller in (6), and in about 10 s by the controller in [9]. The difference in performance arises from a dynamic compensation mechanism in [9] that prevents actuator saturation, as seen in Fig. 2, and thus makes the control conservative and limits the speed of convergence. In contrast, the controller (6) permits simultaneous saturation of several actuators, as shown in Fig. 1, and uses agents' actuation more fully.

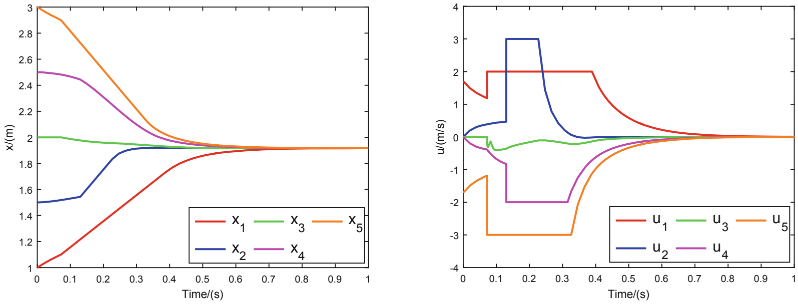

Fig. 1. Coordination of 5 single-integrator agents controlled by (6): positions (*left plot*) and control signals (*right plot*) of all agents.

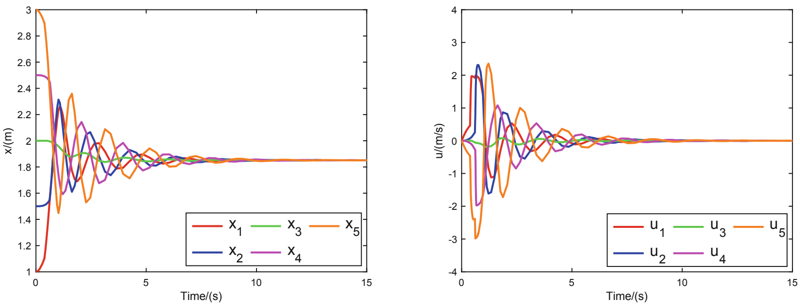

Fig. 2. Coordination of 5 single-integrator agents controlled by the controller in [9]: positions (*left plot*) and control signals (*right plot*) of all agents.

5 Conclusions

This paper has presented a distributed connectivity-preserving coordination controller for single-integrator MAS with bounded actuations. The key steps in the proposed design are: (i) to regard actuator saturation as dynamic scaling of the control; and (ii) to drive the MAS with gradient-based controls derived from local potentials designed to decrease monotonically even when the controls saturate. Then, connectivity preservation results directly from the monotonic decrease of the local potentials. The proposed controller is less conservative than existing distributed connectivity-preserving coordination controllers. With minor modifications, the controller can also guarantee the connectivity-preserving coordination of nonholonomic MAS. Future research will explore sufficient conditions for connectivity-preserving coordination of second-order MAS with bounded actuation.

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